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INVENTORIES AS AN ASSET: THE VOLATILITY OF COPPER PRICES*

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1. INTRODUCTION

The spot price of copper metal is known to be extremely volatile. From 1958 to 1980, the mean monthly average price was \$0.49 per pound (in constant 1967 dollars), while the standard deviation of month-to-month changes in the real price was \$0.06. Claims to copper inventories are traded on the London Metal Exchange (LME), a competitive world asset market. This paper empirically investigates the dynamics of copper spot prices on the LME, focusing on the rate of return to holding copper metal and on the implications of inventory stockouts for this rate of return.

We make one major departure from the standard economic theory of exhaustible resources.[2] Instead of assuming constant short-run marginal cost (SRMC), we incorporate rising SRMC of extraction into the model.[3] This technological assumption provides an economic motive for production smoothing by holding inventories. Equilibrium inventory holding ameliorates rising SRMC by moving extraction to low SRMC time periods. We show that this amelioration is imperfect because of inventory stockouts.

When stockouts can occur, the spot price of a resource reflects transient shocks to its scarcity value as well as permanent shifts. More specifically, the economic motive for inventory holding affects the equilibrium price dynamics of exhaustible resources in two ways. First, as long as positive inventories are being held, their holders should earn the competitive rate of return. Thus, the price of extracted resource should rise at the rate of interest in expected value. This result differs from most recent theoretical studies of exhaustible resources, which conclude that price minus marginal extraction cost should rise at the rate of

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[1] We wish to thank the Sloan Foundation for financial support, and James Rosse, David Starrett and anonymous reviewers for comments. We retain responsibility for all errors.

[2] We are in the tradition of perfectly-competitive resource models beginning with Hotelling [1931]. For surveys of this work, see Dasgupta and Heal [1974], and Peterson and Fisher [1977].

[3] Many exhaustible resource models have treated rising LRMC, as deeper reserves are more expensive to extract. See Hansen [1980] for a recent treatment. But in our model, SRMC may rise above LRMC, leading to production smoothing. In a recent paper, Eswaran Lewis and Heaps [1983] treat exhaustion with a nonconvex production technology. They find nonexistence of competitive equilibrium. Our assumption of rising SRMC departs from the standard model in the opposite direction; we assume the technology is *strictly* convex, while constant SRMC implies only convexity. Thus, issues of existence of equilibrium do not arise in this paper.

interest;[4] but these studies are set in a world without inventories. Second, the possibility of stockouts introduces an asymmetry into the equilibrium price path. In a world with inventory holding, the price path cannot exhibit excess capital gains. All excess capital gains can be arbitraged away. No such arbitrage can prevent excess capital losses. We show that the price of a resource extracted with rising SRMC can take a *perfectly anticipated* fall.[5] This asymmetry in the returns results from the non-negativity constraint on inventories.

2. INVENTORY EQUILIBRIUM WITH RISING SRMC

Let the economy's allocation problem in every time period have two components. First, how much of the stock of reserves of ore should be extracted now? Second, of the extracted stock, what portion should be held in inventories and what portion consumed now?

At every period t, reserves are held below the ground or extracted. Reserves are measured in potential units of extracted resource; R_t is not tons of ore, but tons of metal in the ore. Reserves in period $t+1$ are given by reserves in period t, less extraction in period t, q_t, plus new discoveries made at time t, Δ_t:

$$R_{t+1} = R_t + \Delta_t - q_t. \tag{1}$$

We adopt the accounting convention that holders of reserves sell them to the extracting sector at the time of extraction at price v_t.[6]

At every time period, owners of reserves must decide whether to sell to extractors or hold reserves for future sales. The reserve demand function, R_t^d, is the amount of reserves they choose to hold at time t. (Note: R_t^d is distinct from q_t, the *extractors'* demand for reserves.) Assuming that reserve holders are risk-neutral, the following two equations must hold:

$$R_t^d = \infty \qquad \text{if} \quad E_t[v_{t+1}] > (1+r_t)v_t, \tag{2}$$

$$R_t^d = 0 \qquad \text{if} \quad E_t[v_{t+1}] < (1+r_t)v_t, \tag{3}$$

where $E_t[\cdot]$ is expectation at time t. (2) is the unlimited demand for an asset with an extraordinary expected capital gain, and (3) is the refusal to hold an asset with an expected capital loss relative to other assets.

By equation (2), $E_t[v_{t+1}]>(1+r_t)v_t$ cannot be an equilibrium. Similarly, equation (3) implies that $E_t[v_{t+1}]<(1+r)v_t$ is impossible if owners hold some reserves for the future. Therefore, as long as reserves are held, the expected value of v_t must rise at the competitive rate of interest. The implied equality yields:

$$R_t^d(E_t[v_{t+1}] - (1+r_t)v_t) = 0. \tag{4}$$

[4] See Devarajan and Fisher [1981] and the citations therein.

[5] This result has been derived in the context of production that takes time. See Reagan and Weitzman [1982] and Working [1962].

[6] If extracting firms own reserves, v_t is reinterpreted as the shadow value of reserves. If the extracting sector and the land market are perfectly competitive, these two interpretations are equivalent.

This is a standard finding.

Let the demand for inventories as an asset at time t be I_t^d. The price of the finished refined resource held in inventories is P_t. Note that since v_t is the price of reserves in the ground, not the price of the extracted resource, data on v_t are scarce. However, P_t has an empirical counterpart. Equations analogous to (2), (3) and (4) hold:

$$I_t^d = \infty \quad \text{if} \quad E_t[P_{t+1}] > (1+r_t)P_t \tag{5}$$

$$I_t^d = 0 \quad \text{if} \quad E_t[P_{t+1}] < (1+r_t)P_t \tag{6}$$

$$I_t^d(E_t[P_{t+1}] - (1+r_t)P_t) = 0. \tag{7}$$

Equilibrium in the asset market for inventories implies that equation (7) will be satisfied.

Equation (7), in turn, implies that if inventories are held, expected P_t rises at the rate of interest.[7] Much of the literature on the price paths of exhaustible resources argues that conditions of market power or rising long-run marginal cost (LRMC) in the extracting sector are important to determining the path followed by P_t over time. We conclude, based on (7), that these factors are irrelevant to the short-run price dynamics of resource markets in which a competitive inventory market is observed *and* inventories are actually held.[8] There exists, however, the possibility that inventories could be drawn down to zero. That is, while (5) is impossible, inventory asset demand behavior does not rule out $E_t[P_{t+1}] < (1+r_t)P_t$. This strict inequality may hold during a stockout.

The extracting industry's costs[9] are a function of cumulative extraction to date and the flow of extraction in the current period.

$$C_t = C(q_t, S_t) \tag{8}$$

where $S_t \equiv \sum_{l=0}^{t-1} q_l$ is beginning-of-period cumulative extraction. Extractor profits in period t are given by

$$\pi_t \equiv (P_t - v_t)q_t - C(q_t, S_t). \tag{9}$$

Extractors are price takers, solving

$$\max_q \sum_t \prod_{i=1}^{t} (1+r_i)^{-1}\pi_t \quad \text{subject to} \quad S_{t+1} = S_t + q_t. \tag{10}$$

Let $\beta_t = \prod_{i=2}^{t} (1+r_i)^{-1}$. Then the first-order condition is

[7] Our result that the price will rise at the rate of interest would change if holding inventories were costly. Then the net return would have to equal the competitive return. But there is still no *extraction* cost trend in the price of an inventoried resource.

[8] This also explains attempts by extractors with market power to limit the market for inventories. Producers of oil, for example, often have no-resale provisions in their contracts.

[9] We assume prices allocate production across firms in a cost minimizing way. We are grateful to Michael Hoel for the observation that this is a very strong assumption for this problem. A consistent single index of cumulative extraction need not exist in general.

$$\beta_t(P_t - v_t) - \beta_t C_q(t) - \sum_{\tau=t+1}^{T} \beta_\tau C_S(\tau), \tag{11}$$

where the notation $C_q(t)$ means derivative of time-t costs with respect to q. The second-order conditions are determined by the symmetric matrix, A, with

$$A_{tt} = -\beta_t C_{qq}(t) - \sum_{\tau=t+1}^{T} \beta_\tau C_{SS}(\tau) \tag{12}$$

on the diagonal, and

$$A_{t,\tau} = -\beta_\tau C_{Sq}(\tau) + \sum_{i=\tau+1}^{T} \beta_i C_{SS}(i) \qquad \tau > t \tag{13}$$

off the diagonal. A is negative-definite for a strict optimum for extractors. In a two-period problem, we have

$$A = \begin{vmatrix} -\beta_1 C_{qq}(1) - \beta_2 C_{SS}(2) & -\beta_2 C_{Sq}(2) \\ -\beta_2 C_{Sq}(2) & -\beta_2 C_{qq}(2) \end{vmatrix}. \tag{14}$$

If A is strictly negative-definite, then the extractor's problem has a unique optimum and continuous comparative statics. In these circumstances, extractors will not change their production timing decisions so as to force relative prices at different times into any fixed relationship. Since inventory arbitrage is one-sided, and definiteness of A rules out extractors arbitrage, we have the following theorem.

THEOREM: *Suppose $|-A|>0$. Then there exists an initial stock of reserves and a sequence of demand functions over time such that $P_{t+1}<(1+r_t)P_t$.*

The proof, which is in Appendix 1, proceeds by construction. The demand curve in a given period is shifted out unitl there is an inventory stockout. Further increases in that period's demand then raise its price above future prices.

If $|A|=0$, then the extractors can act as arbitrageurs. But this condition also implies that social cost minimization requires zero inventories. This assertion, proved in Appendix 1, is also quite intuitive. If extraction at different times are perfect substitutes in production, then early extraction is a pure social waste.

Thus, extractive-industry technologies fall into two categories: technologies which imply zero inventories and technologies for which some patterns of demand imply anticipated price falls (APF).

Figure 1 illustrates these results. The heavy right-angle is the asset-market equilibrium, which ensures inventories earn the competitive rate of return (the horizontal segment) or that a stockout occurs (the vertical segment). This right angle curve shows the demand for inventories as an asset. The upward-sloping instantaneous excess supply (IES) curves describe the amount of resource to be placed in inventory, as a function of the refined resource price. The IES curves graph

$$I_t = I_{t-1} - D_t(P_t) + q_t(P_t;\, E_t[P_{t+1}]),$$

where $D_t(P_t)$ is final demand for the resource and $q_t(\cdot)$ is the short-run supply

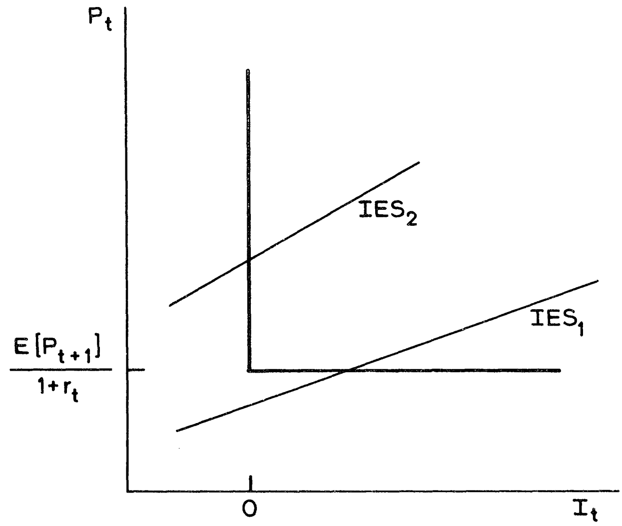

FIGURE 1

curve of the extracting sector as discussed above. In Figure 1, IES_2 corresponds to a higher level of final demand for the resource than IES_1.

Figure 1 shows how the resource market responds to an unanticipated shock. The two IES curves have the same history up to $t-1$. But at time t a shock pushes IES_2 upward, increasing instantaneous demand for the resource. The market consumes the total available stock $(I_{t-1}+q_t)$. The high current price rations the extracted resource available for consumption now. Since the expected future scarcity is less, an APF occurs. If the demand surge does not come in period t, IES_1 holds. No stockout takes place, and the expected price of the resource continues to rise at the rate r.

3. COPPER MARKER EMPIRICAL RESULTS

This section tests our theoretical results using data on the market for refined copper. Copper demand is volatile over time, in part because copper is an important war material and because copper is an input into many durable goods. Copper extraction and refining processes have rising SRMC. On the one hand, both smelting and refining technologies have capacity constraints in the very short run.[10] Also, the lag to increasing mining capacity is several years.[11] Let

[10] Recent innovations have made these processes more efficient in adapting to decreases in demand. (Mikesell, p. 71.) Increases in demand, calling for production beyond normal capacity, are still quite costly.

[11] Although exploration for new reserves is an ongoing process, production in a particular location at a new mine begins at least several years after making the decision to go ahead. (Mikesell, p. 165.)

$$KGAIN_t = \frac{P_t - (1 + r_{t-1}) P_{t-1}}{P_{t-1}}$$

represent the excess capital gain on holding copper. The equilibrium condition based on arguments given above is that

$$E[\text{KGAIN}_t / \Omega_{t-1}] \leqq 0, \tag{13}$$

where Ω_{t-1} is any variable observed at time $t-1$. In this section of the paper, we define operational versions of Ω_{t-1} and test for asymmetry in the equation predicting KGAIN. We no not use data on inventories, though the imperfect inventory data available do not appear to contradict our results.[12]

This test asks whether copper capital gains can be consistently predicted, in the spirit of (tests of) financial market rationality. In our theory, however, some capital gains can be predicted without contradicting risk-neutral market rationality. Thus, we are not testing rationality but are testing for the importance of stockouts.

Before defining exogenous variables (Ω_{t-1}'s) for (13), recall the circumstances in which the price of a resource is expected to fall. During temporary demand surges, we expect binding capacity constraints in extraction. At the end of such surges, KGAIN can be negative in expected value. These circumstances do not suggest specifying Ω_{t-1} as a fixed-lag structure on the capital gains, because such a structure assumes that demand surges have a fixed length. Instead, we posit a variable that is a proxy for the current state of the copper market. This variable should capture the extent to which a temporary demand surge has forced the current spot price up, regardless of when the demand expansion began.

We shall measure this variable in two ways. One method examines how far the spot price is above a price path that rises at the rate of interest. Let P_0 be the price of copper before the sample period and define

$$P_t^* = P_0(1 + r_0)(1 + r_1) \cdots (1 + r_{t-1}).$$

Then our proxy for a current demand surge is[13]

$$PMP_t^* = \frac{P_t - P_t^*}{P_t^*}.$$

[12] Interpretation of the inventory data is difficult because of the existence of "working stocks" and "copper in the process of refining." In an earlier working paper (see Bresnahan and Suslow [1983], especially pp. 25–26 and Tables 5 and 6) we attempted to define a time series of total inventories net of these components. The minima of this series, corresponding to stockouts, occur roughly at the same time as price falls.

[13] The proxy variable PMP* is derived from the theory only to the extent that P* measures some base long-run equilibrium price path. (P* obviously does not allow for innovations during the period.) In all other regards, PMP* is simply one indicator of a departure of the current spot price from its true long-run path. This will be true in the data only to the extent that there have not been permanent shocks to the value of copper within our sample period. For the purpose of rejecting the hypothesis of unpredictability of either capital gains *or* losses, however PMP* is perfectly suitable.

Our tests show that the lags of PMP^* predict falls, but not rises in the price of copper.

Our second proxy incorporates the "producer price" of copper metal, the price quoted by the primary copper producers for delivery of refined copper. Over the long run, the producer price reflects permanent demand shifts, but in the short run, the producer price and the spot price can diverge considerably. While the producer price is tied to the prevailing spot-market price in some respects, many U. S. producers have tried to stabilize the producer price in the face of temporary demand swings. When producers are selling at substantially below the world spot price, we believe that they are delivering on (implicit) forward contracts. For the spot price to exceed the price at which old contracts are being met, the asset value must have increased. If a temporary demand surge caused this increase negative capital gains can be expected in the future. Therefore, the theory perdicts that excesses of the spot price over the producer price should predict falls in the spot price.[14] We define

$$PMPP_t = \frac{P_t - PP_t}{PP_t},$$

where PP_t is the producer price at time t. Again, our tests show that lagged values of PMPP_t predict negative, but not positive, values of KGAIN_t.

In our empirical work, P_t is the spot price (in £/tons) of copper on the London Metal Exchange (LME). Our interest rate, r_t, is the British "call money rate." Both price and interest rate are in nominal £. Their exact definitions are given in Appendix 2. In all regressions, the data are monthly with sample period May, 1958 to December, 1979. P_0 is taken to be the average price over 1957.

Figure 2 graphs the spot price (dots) and producer price (line) over time.

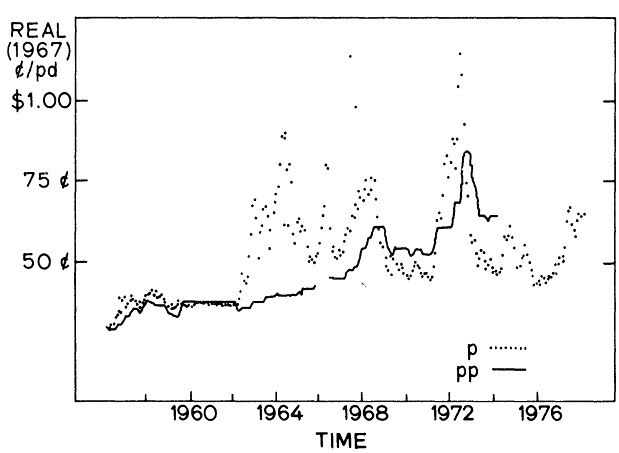

FIGURE 2

[14] There are alternative explanations for why the producer price during our sample period may deviate from the spot price. For one attempt to explain the producer price system as a result of imperfectly competitive behavior, see McNicol [1975].

Although we use nominal values in our empirical work, both series are deflated by an index of industrial input prices for readability. Note that the series depart from one another substantially in the period before 1974, the year that the producer price series stops. Metals trade journals of the mid-seventies often reported that copper producers, in effect, abandoned the producer price system at this time. The suspension of the producer price system during the 1967 U. S. copper strike can also be seen.

3.1. *Empirical Results.* Let $x_t'\beta$ be the predicted-value relation for the capital gain in period t. The x_t are exogenous variables, representing lagged value differences of between short-run and predicted long-run prices. β is a vector of parameters to be estimated. Our upper-truncated model is the nonlinear regression:

$$\text{KGAIN} = \min\,[0, x_t'\beta] + \varepsilon_t$$

where ε_t are assumed independent over time and independently distributed.[15]

In Table 1, we report results for this regression where the first through fourth

TABLE 1
REGRESSION RESULTS: DEPENDENT VARIABLE KGAIN

Exogenous	*Specification*			
	1 lag	*2 lags*	*3 lags*	*4 lags*
PMP^*_{t-1}	−0.249 (0.102)	0.180 (0.099)	0.450 (0.158)	0.551 (0.160)
PMP^*_{t-2}		−0.570 (0.136)	−1.23 (0.329)	−1.47 (0.353)
PMP^*_{t-3}			0.473 (0.189)	0.726 (0.269)
PMP^*_{t-4}				−0.138 (0.127)
Constant	0.185 (0.101)	0.286 (0.093)	0.266 (0.085)	0.299 (0.087)
$\sum_{t-1}^{t-4} (\hat{\beta})$	−0.249 (0.102)	−0.390 (0.093)	−0.307 (0.085)	−0.331 (0.087)
R^2	0.045	0.165	0.190	0.192
DW	1.347	1.57	1.582	1.64
N	260	260	260	260

NOTE: Figures in parentheses are asymptotic standard errors.

[15] Since the indifferentiability of the min[·] raises numerical econometric difficulties, we smooth it in practice. We replace min $[0, x_t'\beta]$ with

$$\frac{\exp[-100^* x_t'\beta]}{\exp[-100^* x_t'\beta] + \exp[100^* x_t'\beta]} \cdot x_t'\beta.$$

This function very closely approximates the min operator. The ratio of exponents is within a millionth of 0 or 1 at every sample point. Its derivative is within a millionth of 0 at those points.

lags of PMP^* provide the x_t'. All four of these lag specifications predict future capital losses at a high significance level. In all four columns, the sum of the lagged coefficients on PMP^* is negative and significant, which is consistent with our theory. If P has exceeded P^* for the last four months, the specifications predict a price fall. The larger the gap between P and P^*, the greater the expected fall. Both PMP^* and KGAIN are percentages: one percentage point increase in the excess of P over P^* implies a third of a percentage point increase in expected capital loss. The positive coefficient on the first lag implies that if P has been above P^* but fell in the last month, a larger capital loss will result. Once the price has started to return to the long-run predicted level, it does so quite quickly. Thus, temporary demand surges end rapidly.

Table 2 reports the regression results with the first through fourth lagged values of PMPP as the exogenous variables. The results are similar, although the coefficients are less precisely estimated in the longer lag structures. We attribute this to the collapse of the producer price system after 1974. Both sets of exogenous variables predict price falls only when the spot price is quite high. None of the eight specifications predicts KGAIN to be anything other than zero unless the spot price has recently exceeded P^* or PP by over 25 percent.

TABLE 2

REGRESSION RESULTS: DEPENDENT VARIABLE KGAIN

Exogenous	*Specification*			
	1 lag	*2 lags*	*3 lags*	*4 lags*
$PMPP_{t-1}$	−0.640 (0.194)	0.351 (0.153)	0.493 (0.168)	0.702 (0.171)
$PMPP_{t-2}$		−0.938 (0.258)	−0.540 (0.307)	−0.627 (0.319)
$PMPP_{t-3}$			−0.442 (0.281)	−0.338 (0.287)
$PMPP_{t-4}$				−0.201 (0.453)
Constant	0.367 (0.235)	0.404 (0.214)	9.380 (0.196)	0.425 (0.223)
$\sum_{t-1}^{t-4}(\hat{\beta})$	−0.640 (0.194)	−0.587 (0.291)	−0.489 (0.194)	−0.464 (0.370)
R^2	0.051	0.132	0.141	0.143
DW	1.35	1.81	1.83	1.83
N	260	260	260	260

NOTE: Figures in parentheses are asymptotic standard errors

While these results are compatible with our theory, other theories must be considered as well. We first more precisely test the proposition that the asymmetry in the prediction of capital gains and losses is driving our results. Let the regression specifications be

$$KGAIN_t = \begin{Bmatrix} x_t'\beta, & x_t'\beta \leq 0 \\ \alpha x_t'\beta, & x_t'\beta > 0 \end{Bmatrix} + \varepsilon_t,$$

where α is a new parameter to be estimated.[16] Our earlier specification required $\alpha = 0$; allowing an $\alpha > 0$ permits the predicted-value relation to be non-coincident with the x-axis. Figure 3 illustrates this more general case. Figure 3a presents our original specification. For small α, as in Figure 3b, the predicted-value relation is similar to the original. If in fact $\alpha > 1$, a case shown in Figure 3c, the asymmetry would run in the opposite direction. An estimated α near zero and significantly different from 1 will confirm the form of the asymmetry we suspect, while other α's permit a broad class of asymmetries.

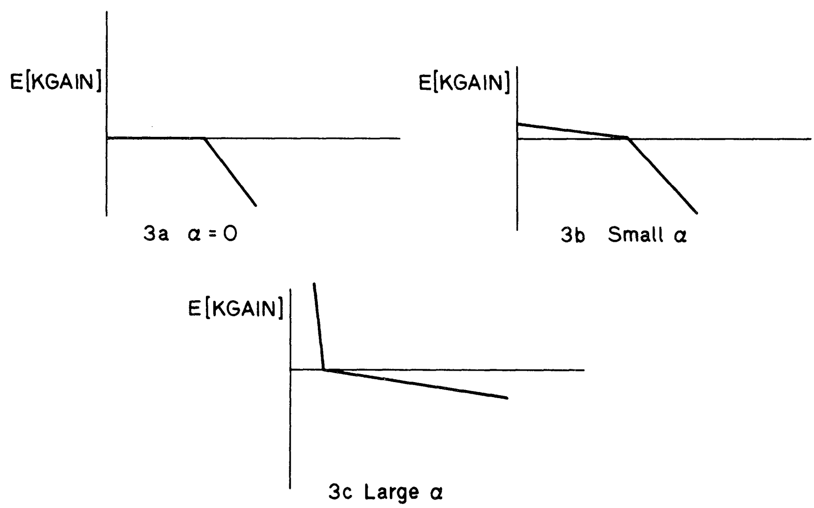

FIGURE 3

Table 3 shows estimates for this specification when x_t contains three lags of *PMP** or PMPP. In these estimates, α is quite small and is not significantly different from zero. The estimated β parameters are very similar to those in the specification with $\alpha = 0$.

Finally, either set of exogenous variables will reject the hypothesis that $\alpha = 1$. We conclude that lagged values of *PMP** or PMPP can predict capital losses but not capital gains.[17]

[16] As before, we avoid the problem of indifferentiability by smoothing

$$\frac{\alpha \exp[100^* x_t'\beta] + \exp[-100^* x_t'\beta]}{\exp[100^* x_t'\beta] + \exp[-100^* x_t'\beta]} \cdot x_t'\beta.$$

Again, this function approximates the one in the text very closely.

[17] Earlier empirical studies did not allow the standard-trended random walk result to fail, and they have not led to a clear characterization of copper price movements. Labys, Rees, and Elliot [1971] investigated copper prices over time and failed to find any departures from a random walk using purely time-series methods.

TABLE 3

TESTING FOR ASYMMETRY: DEPENDENT VARIABLE KGAIN

Exogenous	*Specification*	
PMP^*_{t-1}	0.463 (0.159)	
PMP^*_{t-2}	−1.24 (0.325)	
PMP^*_{t-3}	0.486 (0.186)	
$PMPP_{t-1}$		0.594 (0.136)
$PMPP_{t-2}$		−1.65 (0.419)
$PMPP_{t-3}$		0.721 (0.503)
Constant	0.251 (0.077)	0.298 (0.087)
α	0.062 (0.033)	−0.014 (0.160)
$\sum_{t-1}^{t-3} (\hat{\beta})$	−0.291 (0.085)	−0.335 (0.124)
R^2	0.191	0.146
DW	1.62	1.83
N	260	260

NOTE: Figures in parentheses are asymptotic standard errors.

The most important alternative explanation for our empirical results is the following: our lagged variables may be able to predict capital gains because they proxy for some risk "factor." Our regressions might predict expected capital gains if securities prices contain a risk premium. In particular, the Arbitage Pricing Theory (APT) of Ross develops an elegant analysis in which several risk factors are reflected in asset prices.[18] Our strong finding of asymmetry is difficult to explain in this context. We find that high values of PMP^* imply smaller than usual returns to holding copper. If they are proxies for a risk factor, they must predict either a lower variance in copper prices or a closer covariance of copper prices with the return on other assets. In fact, the variance of copper prices also appears to be high when PMP^* or PMPP is large. This added risk is probably not diversifiable. Hence, the APT provides no obvious criticism of our findings.

We can also test the hypothesis of risk premiums by estimating an equation we do not expect to have predictive power. For purely financial assets, our theory predicts no asymmetry in the distribution of returns. On the other hand, if the lagged values of our chosen exogenous variables are proxies for some risk factor, that factor should affect both copper metal and owners of copper reserves. Our new equation has the same form as before, except that the dependent variable is

[18] See Roll and Ross [1980] for an exposition of this theory and its empirical implications.

now the return on holding the stock of Kennecott Corporation, a copper mining firm, rather than the return on holding copper metal. In Table 4, we report the results of specifications which use the return on this purely financial asset as the dependent variable. All have three lags of PMP^* as independent variables.[19] If our findings merely reflect a risk premium this equation should be statistically significant.

TABLE 4
TESTING FOR ASYMMETRY:
DEPENDENT VARIABLE RETURN ON HOLDING KENNECOTT STOCK

Exogenous	*Specification*		
	OLS	Truncated *at Zero*	Bends *at Zero*
PMP^*_{t-1}	−0.023 (0.048)	−0.058 (0.101)	−0.159 (0.182)
PMP^*_{t-2}	−0.0099 (0.077)	0.026 (0.141)	0.163 (0.236)
PMP^*_{t-3}	0.021 (0.040)	−0.106 (0.126)	−0.214 (0.211)
Constant	0.010 (0.006)	0.110 (0.111)	0.184 (0.189)
α	—	—	0.495 (0.178)
$\sum_{t-1}^{t-3}(\hat{\beta})$	−0.012 (0.018)	−0.138 (0.112)	−0.210 (0.178)
R^2	0.005	0.004	0.006
DW	2.114	2.158	2.178
N	260	260	260

NOTE: Figures in parentheses are asymptotic standard errors.

These equations are completely unable to predict the stock price. They show no evidence of asymmetry: we cannot reject the hypothesis that $\alpha=1$. This result supports our earlier interpretation of the empirical findings. Assets which have uses in the real side of the economy, like metals, are different from purely financial assets. Events on the real side, such as demand or supply shocks, can affect prices more directly. In this theory, cyclical as well as permanent phenomena are reflected in metals prices.

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[19] One might object to this evidence on the grounds that before 1974, Kennecott was committed to sell at the producer price. We reestimated the two specifications of Tables 4 and 5, Column 3, over the subsample periods — May 1951, December 1973, January 1974, and December 1979. The likelihood-ratio test for equality of the two sets of coefficients fails to reject in all three cases.

APPENDIX 1

1.1. *Proof of Theorem*

In this economy, competitive equilibrium supports the social optimum. The objective function is, thus, total benefits (consumer surplus, denoted by $CS(x_t)$) minus total costs ($C(q_t, S_t)$). Without loss of generality, we assume price rises at the rate of interest from period two onward. An arbitrarily high function $CS(x_1)$ in period 1 will lead to price falling by period 2.

Assuming that marginal rent rises at the rate of interest from period 2 forward, we can take the value function for $t=3, 4,\ldots$ as given at time 3. Call this summary value $V_3(R_3)$. Let initial reserves be large enough so that the economy never faces a reserve constraint in either of the first two periods.

The problem becomes

$$\begin{aligned}\max_{\{x_1, q_1, x_2, q_2, I_2, S_2\}} L = {}& \alpha CS(x_1) - C(p_1, S_1) - \beta_2[CS(x_2) - C(q_2, S_2)] \\ &+ \beta_3 V_3(\bar{R}_3 - S_2 + I_2 - x_2) - \lambda(I_2 - \bar{I}_1 - q_1 + x_1) \\ &+ \mu I_2 + \psi(S_2 - \bar{S}_1 - q_1),\end{aligned}$$

where $\beta_2 = \dfrac{1}{1+r_2}$ and $\beta_3 = \dfrac{1}{(1+r_2)(1+r_3)}$ are discount factors, and S_t and I_t are beginning-of-period values ($\bar{S}_1$ and $\bar{I}_1$ given). Demand assumptions are sufficient to ensure $CS_x > 0$, $CS_{xx} < 0$, $V_3' > 0$, $V_3'' < 0$.

Note that

$$V_3(\bar{R}_3 - S_3 - I_3) = V_3(\bar{R}_3 - (S_2 + E_2) + (I_2 + E_2 - x_2)) = V_3(\bar{R}_3 - S_2 + I_2 - x_2),$$

using inventory and stock constraints for period 3.

I_2 has an inequality constraint, implying that the first-order condition on μ is:

$$\mu \geq 0, \quad I_2 \geq 0 \quad \text{and} \quad \mu I_2 = 0. \tag{A.1}$$

The parameter α can change the value of consumption in period 1.

LEMMA 1. *Either $I_2 = 0$ for all α or there exists $\bar{\alpha}$ such that $\alpha > \bar{\alpha}$ implies $I_2 = 0$ and $\alpha < \bar{\alpha}$ implies $I_2 > 0$.*

PROOF. Under the assumption $\mu = 0$ and $I_2 > 0$, the comparative statics of equilibrium are determined by an 8×8 system.

Cramer's rule on this system yields:

$$\frac{dI_2}{d\alpha} = \frac{CS_x^1\{\beta_2 CS_{xx}^2 + \beta_3 V_3''\}[(\beta_2)^2(C_{SS}^2 C_{qq}^2 - (C_{qS}^2)^2) + \beta_2 C_{qq}^1 C_{qq}^2]\}}{\Delta_1}.$$

$\Delta_1 > 0$ by second-order conditions. By our demand assumptions, $CS_x^1 > 0$, $\beta_2 CS_{xx}^2 + \beta_3 V_3'' < 0$. The term in square brackets is the determinant of (14); hence, it is nonnegative by the second-order conditions. If it is zero, $I_2 = 0$ by the discussion below (14). If t is positive, $dI_2/d\alpha > 0$. Q.E.D.

LEMMA 2. *If* $I_2 = 0$, *then* $\frac{d\mu}{d\alpha} > 0$.

PROOF. Under the assumption $\mu > 0$ and $I_2 = 0$, the comparative statics of equilibrium are determined by a similar 7×7 system.

$$\frac{d\mu}{d\alpha} = \frac{CS_x^1\{\beta_2 CS_{xx}^2 + \beta_3 V_3'')[(\beta_2)^2(C_{qq}^2 C_{SS}^2 - (C_{qS}^2)^2) + \beta_2 C_{qq}^1 C_{qq}^2\}]}{\Delta_2} + \frac{2(\beta_2)^2 CS_{xx}^2 C_{qq}^2 \beta_3 V_3''}{\Delta_2},$$

where $\Delta_2 < 0$ by the second-order condition. Assume a sequence of low demands after period 2 so that V_3'' becomes arbitrarily small. Then, positivity of the term in square brackets implies $\frac{d\mu}{d\alpha} > 0$. Q.E.D.

By increasing α we can make μ strictly greater than zero; in this condition, the economy would like to hold strictly negative inventories. Of course, μ strictly greater than zero implies that the price falls from period one to period two.

APPENDIX 2

2.1. *Data*

Our price data are monthly average spot prices of electrolytic wirebars, a standardized copper commodity, on the London Metal Exchange (LME).[1] The LME provides both a world competitive spot market and a hedging facility for copper and other nonferrous metals. Approximately ten LME warehouses in Europe hold copper for delivery. The minimum LME contract is for 25 tons and contract lengths range from spot to ninety days. Although the LME is not primarily a market for physical deliveries of copper,[2] most long-term contracts between refiners and fabricators specify prices or pricing formulas based on the LME.[3] Thus, the LME price can be considered the world spot price for copper.

The interest rate series is the end-of-month "Call Money Rate."[4] Some British sources refer to the Bank of England's rediscount rate as a call money rate, but we use the OECD definition. Experiments with the consols interest rate series had no effect on our results. Both the price and the interest rate series are nominal, with pounds sterling the currency of record.[5]

[1] Source: ABMS.

[2] In 1968, for example, futures contracts on the LME represented about 30 percent of world consumption in that year while actual LME deliveries amounted to only two percent. (Banks, p. 41.)

[3] Mikesell, pp. 79–84.

[4] Source: OECD Main Economic Indicators.

[5] When the ABMS reports cents per pound rather than pounds sterling per ton, we use the spot exchange rate to convert. Source: OECD Main Economic Indicators.

Returns on the stock of Kennecott Corporation come from the Center for Research on Securities Prices (CRSP) at the University of Chicago monthly stock price file. We used the monthly return including dividends as defined by CRSP. This percent return is nominal.

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