Heterogeneous Applications and Platform Market Stability: Mobile Apps *

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Abstract

We introduce a new version of the classical model of equilibrium stability and tipping in platform markets. We retain the positive feedback elements: Users choose platforms in part for the applications available on them, and developer profits are increasing in the number of users on a platform. We incorporate heterogeneity across applications in attractiveness to users: Some apps are stars, while others are less popular. Application heterogeneity is a characteristic of consumer demand for games, information, and entertainment products, and thus the most important modern platform industries. Application heterogeneity changes platform market stability analysis with surprising results. Star-dominant application heterogeneity implies *stability of divided equilibrium* for large platforms in large markets. This contrasts with the familiar prediction with undifferentiated applications, tipping away from unstable divided equilibrium to a dominant platform. Our model also predicts instability and tipping in smaller markets or with smaller platforms in large markets. All our predictions arise from the same economics. Star applications are inframarginally supplied to platforms with a large installed base, dampening the positive feedback loop. Our model predicts that the same dampening will not arise at smaller installed bases where star applications' supply is marginal. We estimate our model on data from the US smartphone application market. Both user demand for applications available on a platform and developer supply of applications to platforms show star-dominance as a first order phenomenon. Stability bounds show that this is consistent with the longstanding stable divided US smartphone platform duopoly. We also show that the same estimates are consistent with the more tippy aspects of these markets. Our estimates predict the observed tendency of smaller platforms (e.g., Windows Mobile) to tip out of the US market. Smartphone markets outside the US show considerable tippiness, and our estimates predict tippiness in markets smaller than the US. The success of applications heterogeneity as a new element of platform stability analysis suggests the value of adopting platform theory to incorporate new assumptions about developers and users.

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I Introduction

Platform markets often tip to a dominant platform, with applications developers tending to supply the platform with the most users and users tending to choose the platform with the most applications. Economists explain this with a now-classical theory of platform market stability.[1] The recent extension of the platform industrial organization to consumer mass markets has led to a surprising mix of market structures. The smartphone market in the United States, the largest applications development platform market of all time, has persisted in an equilibrium approximately equally divided between iOS and Android.[2] Other, smaller, national markets show more tendency to tip. Even in the large US market, platforms with a smaller installed base, like Windows Mobile (WinMo), have tipped out. These 21st century consumer-oriented applications markets call for a new approach to platform market stability analysis.

In this paper, we provide an economic explanation of these phenomena with both theoretical and empirical analysis. The core theoretical idea is that applications heterogeneity in attractiveness to users changes platform stability analysis. In our industry, applications are heterogeneous in how attractive they are to users: some are *stars*, demanded by many or even most users, while other applications are mundane, attracting only a modest amount of user demand. Applications are heterogeneous in their attractiveness to users for a variety of reasons. Some have powerful network effects themselves, like Facebook or Twitter. Others help consumers use an already-popular service, like United Airlines or Citibank. For our purposes, what is important is *that* a particular application is very attractive to consumers, not *why* it is. Supply of a star application to a platform increases the value of that platform to users more than supply of a mundane application. At the same time, the developer of a star application earns more economic return for a given installed base than does a mundane application. A star application's threshold installed base for profitability on a platform is lower, and the supply of the most attractive applications is inframarginal for a platform with a large installed base of users. When user installed base changes, the supply of the most attractive applications to platforms changes little, dampening the strength of the platform positive feedback loop.

The elasticity of applications supply to platforms with respect to platform installed base is a core component of the positive feedback loop. Quantitatively, the supply elasticity depends on depends on the shape of the distribution of applications attractiveness and on the installed base. Because the supply of smartphone applications consists in large part of mass-market consumer information, media, and entertainment, products, we will be particularly interested in a star-dominant distribution of application heterogeneity. With a star-dominant distribution, most of the aggregate attractiveness of applications comes from stars,

1. See Farrell and Klemperer (2007) and Rysman (2009) for reviews of the platform literature, and see section II below for further citations.
2. See industry structure discussion at Figure 4 and below.

not from the (potentially more numerous) mundane applications. In Section III, we formalize the idea of a star-dominant distribution of applications attractiveness and show precise versions of this intuition. Star-dominant product heterogeneity, while new in platform economics, is familiar in media and entertainment mass markets.[3]

Inelastic application supply to all platforms at a candidate equilibrium pushes it toward stability. Only if consumer platform choice is highly elastic with respect to applications availability will the equilibrium be unstable. This, plus star-dominance, is the key to our theoretical results about platform market stability. First, in large markets, an evenly divided platform market equilibrium can be stable with unstable equilibria at less evenly divided platform market shares. This result reverses the "folk theorem" of platform models with indirect network effects and traditional assumptions about representative applications, in which the divided equilibrium is unstable and dominant-platform equilibria are stable. Applications heterogeneity changes these results *without* dropping the positive feedback elements of platform markets. Second, with the same supply of applications and demand for applications and platforms, divided platform market equilibrium will be more stable in a large market and more unstable in a small one. The reverse result holds under traditional assumptions. Finally, a platform market equilibrium with two (or more) approximately equally-sized platforms and another smaller one can be unstable even when a divided platform market equilibrium would be stable. All these surprising results require star-dominance in the application heterogeneity and that user platform demand not be too elastic with respect to available applications. In our empirical results, we shall measure the shape of the smartphone application heterogeneity and the elasticity of supply of applications to platforms and bound the elasticity of user demand for platforms.

We empirically estimate the distribution of heterogeneity in applications attractiveness to users and a linked platform supply model for applications developers in the US smartphone market. Our goals are to implement a quantitative version of our theoretical model and to test the crucial assumption of a star-dominant application heterogeneity distribution. We assemble a dataset in which an observation is an application. The cross-section dataset contains information on all the economically significant iOS and Android applications in the US market. Section IV has details on the sources of our data: a commercial product measuring user applications demand on platforms and our own collection of information about applications and developers.

Estimating the distribution across applications of attractiveness to users is conceptually simple but econometrically complex. An applications' attractiveness to consumers on a platform is only observed if it is 1) supplied for that platform and 2) included in our baseline commercial sample.[4] Accordingly, we

3. See review articles by Sorensen (2017) and Waldfogel (2017) as well as further discussion of the literature in Section II.
4. It is not practical to avoid selection of a sample of applications in our industry. While there are millions of applications

estimate a joint model of observed application attractiveness (conditional on being observed on a platform), applications supply to platforms, and applications' inclusion in the sample. The distribution of attractiveness underlies all these as a primitive. We also gather additional data on supply in order to make the selection model sharper.

The economics of developer supply are like those of market entry, with access to groups of customers driving the profitability of supplying one or more platforms.[5] With heterogeneity, a developer's profits on a platform depend on their application's attractiveness as well as on the installed base of users. Our model allows the attractiveness of a given application to differ across each of the major platforms, iOS and Android, and estimates the degree to which they are dependent. Our market, like many consumer-facing media and entertainment markets, has uncertainty about product success *ex ante*.[6] We model this uncertainty explicitly. A related modeling issue concerns the set of potential suppliers to each platform. Like many entry studies starting with Berry (1992), we only observe a potential entrant if it is an actual entrant in the market of interest or in an adjacent market. In our industry, the adjacent market is the other platform. Modeling this leads us to solve a longstanding problem in entry models.

To move toward a quantitative realization of the stability analysis, our empirical model adds a number of elements. Some of these elements are general to mass market consumer goods industries, including discovery of applications by users and, relatedly, gaps between a developer's forecast of an application's attractiveness at the time of entry and its ultimate market importance. These forces are important for quantification because they loosen the connection between applications heterogeneity in demand and in supply. Other elements of our empirical model are related to platform industries with competing platforms, including the possibilities that the distribution of application attractiveness is different on different platforms and that application profitability varies across platforms. These forces are important for quantification because they weaken the sense in which an applications development platform is, from a star developer perspective, just another market with some installed base of customers to enter. Finally, we account for the highly heterogeneous nature of mobile applications and developers. For example, entrepreneurial Rovio's "Angry Birds" games and long-established Citibank's mobile banking application are both attractive to users, but we do not want to assume *ex ante* that their costs or the value they place on additional customers on a platform are the same.

Our estimates show, as is also clear in an examination of the raw data, that the assumption of undiffer-

on each platform, the bulk of user demand is for a modest number of applications (Bresnahan, Davis and Yin (2015)). We use a commercial sample of applications that are economically important on either iOS and Android.

5. Profit is an economic concept here, not an accounting one. In our industry, some applications earn profit literally by being sold or, more typically, through "in app purchases." Others are advertising supported. Others are complements to products sold by the firm, such as air travel and an airline's mobile app.

6. See Aguiar and Waldfogel (2018) and Hendricks and Sorensen (2009) for empirical examinations of particular industries, and Sorensen (2017) and Waldfogel (2017) for broader reviews of the literature.

entiated applications is not tenable. Application heterogeneity is a first order phenomenon in both demand and supply. The quantification in our estimates, together with our update of the theory, leads to a series of results about platform market equilibrium. The distinct supply behavior of star applications and more mundane ones is central both to the stability analysis of the US market and to an examination of how stability would change if there were a smaller platform (like Windows Mobile) or if the analysis were applied to a smaller economy.

Our estimates (Section VI) show that the density function of application attractiveness for each of the iPhone and Android platforms are similar, and each has a star-dominant shape. The elasticity of supply of applications to platforms with respect to the installed base of users is quite low, around 0.02 for either iOS or Android. At the historical US duopoly, in other words, the applications supply part of the platform positive feedback loop is quite damped. The implications for stability are seen in a bound on the demand elasticities. To find the bound, we assume demand is logit, so one parameter determines all demand elasticities. The elasticity of iPhone users' demand for iPhones with respect to available applications would need to be over 25 for the observed US duopoly to be unstable. In contrast, if we examine supply at smaller installed bases for both platforms (a within-sample analysis), we get much smaller bounds. The implication is that a market about 1/6 the size of the US would have an (in)stability index about an order of magnitude larger than the US, i.e., would be much closer to tippyness. Similarly, a three-platform equilibrium, formed by adding a third, smaller platform to the historical US duopoly, would be much closer to tippy. Our model predicts both the stability of the US duopoly and tippiness of an equilibrium with at least one platform having a significantly smaller installed base. We shall see that the model's economic explanation of stability of equilibrium with large platforms implies, rather than contradicts, instability with significantly smaller platform installed base.

II Prior Work

We draw on two previously unlinked literatures, platform stability analysis and mass media markets.

The economic impact of network effects is a well-studied problem theoretically with papers dating to Rohlfs (1974). The competition between platforms is another rich theoretical literature, with seminal contributions from Katz and Shapiro (1985) and Farrell and Saloner (1985). Farrell and Klemperer (2007) provide a deep review of the literature on network effects, with more emphasis on the theory. The theoretical literature analyzes the sources and implications of network effects (where we make our contribution) and of their normative implications. Rysman (2004) provides an overview of both empirical and theoretical work on platform markets. We do not draw out the normative implications of our work, focusing instead on the positive economics of stability.

A number of papers consider the possibility that platform competition could be dulled by forces that offset network effects and positive feedback, emphasizing the economic relationships between users and developers. One structure offsets the positive externalities of indirect network effects by adding negative externalities among users, such as congestion, or among developers, such as competition to sell similar applications (e.g., Ellison and Fudenberg (2003)). Another structure, closer to our approach, assumes that platforms themselves are differentiated products either to developers or to users or to both (e.g., Church and Gandal (1992), Cantillon and Yin (2008)). One form of differentiation is related to user preference heterogeneity; if some users value the number of applications on the platform more than others, platform markets can be vertically differentiated with a many-application and a few-application platform in equilibrium (e.g., Gabszewicz and Wauthy (2014)). None of these treatments, however, embody our model of heterogeneity in attractiveness.

There are a number of empirical papers which examine platform industries with a focus on tipping and tippiness. If the market has tipped to a dominant platform, it is hard to find variation in the installed base or in the number of applications in-sample. Rysman (2004) is one of the few empirical studies of platform industries which observes variation in industry structure (across local markets). More papers examine the process of moving toward an equilibrium (e.g., Church and Gandal (1992), Gandal, Kende and Rob (2000)). In this vein, Augereau, Greenstein and Rysman (2006) distinguish between a process of "coordination" on a common standard versus moving to a divided equilibrium with "differentiation" in standards, with one side of the market preferring the differentiation to impose switching costs on the other side. Another approach is to look at performance indicia of the platform to infer network effects. If an incumbent platform performs worse than an entrant, for example, but nonetheless maintains a high market share, one might infer that network effects are holding back the entrant platform. This approach leads to a finding of "endogenous platform differentiation" rather than a clear advantage for the incumbent platform in Hendel, Nevo and Ortalo-Magne (2009) and to a finding of network advantages in Brown and Morgan (2009).

The other foundational literature for us studies consumer media markets. Hendricks and Sorensen (2009) study music sales, noting that product demand is distributed so that most of industry profit is earned by only a few products. Further, they conclude that the tendency for much of sales to come from only a few products is heightened by consumers' difficulty in learning about products. Aguiar and Waldfogel (2018) note that uncertainty by producers about a product's success at the time of investment plays a similar role and that better *ex ante* signals of demand, which they call "quality predictability," lead to changes in supply that expand demand. Similar findings about the shape of demand and the role of information have been found for books (Chevalier and Mayzlin (2006), Sorensen (2007)), motion pictures, and other consumer media industries. Sorensen (2017), reviewing studies of a number of industries, notes what we call a star-dominant distribution of success for a wide range of consumer product industries and discusses how incomplete consumer information plus search leads, in some circumstances, to even greater spread in demand heterogeneity at the product level. Waldfogel (2017) notes that the return to digitization in many media markets often arises because lower costs create more chances for the creation of very high quality products, i.e., star products at the top of the distribution of applications heterogeneity. Whether either of these ideas – the importance of a small number of products in aggregated demand and the importance of information to consumers and producers about products – applies to our industry is an empirical question.

The kind of supply behavior we study is familiar in empirical models of entry.[7] Our setup shares a long-standing "potential entrants" problem with entry models which, like Berry (1992), identify a list of potential entrants into one market as the actual entrants into other markets.[8] If a firm's profit in one market is not independent of its profit in another, the list of potential entrants is not exogenous. This problem certainly applies in our context: we use actual entrants into platform p as the potential entrants into platform p'. Since we have a model of entry into both platforms, we correct for selection without adding any new "selection" parameters.

The particular supply behavior we study is inframarginal multihoming by star developers. As we shall see in the next section, the "inframarginal" is very important here, unlike the "multihoming." Accordingly, our analysis is only loosely linked to the rich literature on multihoming.[9]

III Application Heterogeneity and Platform Market Tipping

The classical indirect network effects model has two positive feedback elements: applications developers are more profitable on platforms with more users while users value a platform in part for its applications. In this section, we first examine the traditional version of this model, with its folk theorem result that divided platform market equilibrium is unstable even with symmetric user demand and developer supply. We then introduce a new version with applications heterogeneous in their attractiveness to users, and thus in the value they contribute to a platform and in their supply behavior. We examine conditions for stability of divided equilibrium in this new version. We examine the relationship between stability and market size in both versions.

7. See Berry and Reiss (2007).

8. Some entry models like those of Bresnahan and Reiss (1990) and Seim (2006) identify a set of market niches rather than a set of potential entrants and thereby avoid this problem.

9. Corts and Lederman (2009) study developer supply in the game console industry, emphasizing the role of multihoming in limiting tipping. Venkataraman, Ceccagnoli and Forman (2017) analyze multihoming in a very different context where strong links between platform provider and complementors flow through shared human capital. Grajek and Kretschmer (2012) examine "critical mass" in mobile telephony itself.

III.A The Folk Theorem

We start with the traditional version in which undifferentiated applications select between two platforms. Strong enough positive feedback leads to tippiness, with an unstable divided equilibrium and stable equilibria with a dominant platform. (Proofs of the propositions in this section are in the Appendix.) This result underlies much thinking about platform markets and has become a folk theorem. This version of the model is also always less stable in larger markets.

The profit for application a on platform p depends on the installed base of users, U_p:

$$\Pi_{ap} = \mu U_p - C_p + \epsilon_{ap}. \tag{1}$$

Each application's profit increases with the installed base at rate $\mu \geq 0$. Developers are heterogeneous only in fixed profit; it is as if ϵ_{ap} is a negative shock to *fixed* costs C_p. This traditional version of the classical model captures an essential feature of application supply to platforms. Software has high fixed costs and low marginal costs, so developers tend to prefer a platform with a larger installed base.[10] Each developer chooses the platform with higher Π_{ap}. Let $F_a()$ be the strongly unimodal and symmetric cdf of $\epsilon_{a2} - \epsilon_{a1}$, and let $\Delta_\pi \equiv \mu(U_1 - U_2) - C_1 + C_2$ be the difference in mean profitability between platforms. The developer platform supply equation is

$$N(U) = \left[F_a(\Delta_\pi(U)) \;\; 1 - F_a(\Delta_\pi(U))\right]. \tag{2}$$

The second positive feedback element is that users value platforms in part for the applications available on them. In this version applications are undifferentiated and user platform demand depends on the number of available applications on each platform. Let $\gamma \geq 0$ be users' valuation of applications, and γ_p be the mean intrinsic value of platform p to users, so the utility of choosing platform (p) for user (u) is

$$V_{up} = \gamma N_p + \gamma_p + \epsilon_{up}. \tag{3}$$

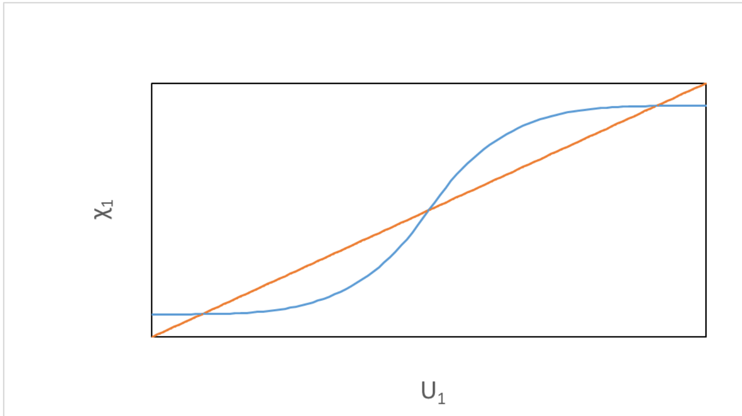
We normalize the mass of applications to 1, but to examine market-size effects we let the mass of users, U_M, vary. Let $\Delta_V(N) = \gamma(N_1 - N_2) + \gamma_1 - \gamma_2$ and let $F_u()$ be the cdf of $\epsilon_{u2} - \epsilon_{u1}$ (heterogeneity across individual users) which is smooth, strongly unimodal, and has infinite support. User platform demand is

$$U(N) = U_M \times \left[F_u(\Delta_V(N)) \;\; 1 - F_u(\Delta_V(N))\right]. \tag{4}$$

Finally, an equilibrium U_1^e is fixed point of the mapping $\chi_1(U_1) \equiv U(N(U_1, U_M - U_1))_1$ and an equilibrium

10. The pricing of apps is pushed into the background here, but most models have a positive equilibrium (after pricing) marginal profit as the size of the installed base increases.

Figure 1: The Geometry of the Folk Theorem



is stable if the (in)stability index $\chi_1'(U_1^e) < 1$, where[11]

$$\chi_1'(U) = 2 * \gamma * f_u(\Delta_V(N(U))) * U_M * \mu * f_a(\Delta_\pi(U)). \tag{5}$$

The index, plus the geometry of χ_1 (a function from a closed interval on $\mathbb{R}^1$ to itself) permits simple demonstration of a number of points (more details in Appendix). An equilibrium is a point where χ_1 cuts the 45° line. There will always be at least one equilibrium, and there will always be one stable equilibrium, a point at which χ_1 cuts the 45° line from above. If demand and supply are symmetric, i.e., $\gamma_1 = \gamma_2$ and $C_1 = C_2$, there will always be a divided equilibrium with $U_1 = U_2$ and $N_1 = N_2$. Assuming symmetry, the equilibrium correspondence can take on only two forms: one with a unique, divided equilibrium, and one with three equilibria, as in Figure 1, i.e., an unstable divided equilibrium and two stable dominant-platform equilibria. In one of these, with high U_1 and N_1 but low U_2 and N_2, users choose platform 1 because developers do, and developers choose platform 1 because users do. The other stable equilibrium is the opposite, with high U_2 and N_2 because of positive feedback. The three-equilibria scenario is the core of the folk theorem.

It is also simple to see the condition determining whether there is a unique, stable, divided equilibrium or instead three equilibria. Expression (5) will be greater than 1 at the divided equilibrium if $\gamma * \mu * U_M > 1/(2 * f_a(0) * f_u(0))$. In that case, there are three equilibria, following the pattern of Figure 1. If the opposite inequality holds, there is a unique, stable divided equilibrium. The economics have two elements. If $\gamma * \mu$ is sufficiently small, network effects are modest and platform market equilibrium is unique. With larger network effects there are multiple equilibria. Perhaps less well known, U_M plays exactly the same role

11. Equivalently, an equilibrium U^e of the system defined by (2) and (4) is a fixed point of the mapping $\chi(U) \equiv U(N(U))$. An equilibrium is stable if the real part of all eigenvalues of $J_\chi(U^e)$, the Jacobian of χ, are less than 1 in absolute value. In the two-platform case considered here with user single-homing, the Jacobian has one non-zero eigenvalue equal to the slope of χ_1. In the Appendix and in some of the analysis reported in Section VI.C, we have more than two platforms and must use the matrix version.

as γ or μ; modeling developers as having fixed plus constant marginal costs makes a larger installed base equivalent to per-customer profit μ.

Tipping to a dominant platform, arbitrarily either 1 or 2, and unstable divided equilibrium, are the core positive economics results of the traditional version. These results require not only the assumption of indirect network effects but also the representative application assumption.

III.B Application Heterogeneity

We consider a form of applications heterogeneity prevalent in many modern consumer-oriented platforms. Some applications are stars, highly attractive to consumers and able to generate higher developer profit for a given user installed base. Other applications are less attractive and less profitable. The supply behavior of applications to platforms changes with this assumption: even at a low installed based, developers profitably supply the applications that really attract users. In our industry the range of heterogeneity, measured by per-customer demand, is very wide. Stars like "Angry Birds" have tens of millions of US users, while "Bird Sounds Ringtones" has hundreds of thousands. To see how this heterogeneity affects the economics of platform stability, we construct a new version of the classical platform stability analysis.

Each application, a, has an index of attractiveness, r_a, with $0 \leq r_a \leq 1$. The index enters three places: demand for the application, the application's contribution to the attractiveness of a platform to users, and the developer's return to supplying the app to a platform. First, application demand: If application a is available on platform p, the number of users of the platform demanding it is

$$Q_p = r_a U_p. \tag{6}$$

Second, we make the conforming assumption that higher r applications make a larger contribution to the attractiveness to users in the *platform* market. Let all applications with $r_a > \widehat{r}_p$ be available on platform p. The index of application attractiveness on platform p, called v_p, is the total, across all applications available on p, of their attractiveness:

$$v_p = \int_{\widehat{r}_p}^{1} t f_r(t) dt, \tag{7}$$

where $f_r()$ is the density function of the distribution of r_a across apps. User platform demand otherwise is the same as (4), but the definition of Δ_V is now $\Delta_V = \gamma(v_1 - v_2) + \gamma_1 - \gamma_2$, so:

$$U(v) = U_M \times [F_u(\Delta_V(v)) \;\; 1 - F_u(\Delta_V(v))]\,. \tag{8}$$

Denoting the application's per-customer profit once again by μ, and setting the fixed costs of writing an app for platform p to C_p, the total return to supplying an app of attractiveness r_a to a platform with U_p users is

$$\Pi_{ap} = \mu r_a U_p - C_p. \tag{9}$$

The applications supplied to a platform are those with r_a above a breakeven threshold, $\widehat{r}_p$, where

$$\widehat{r}_p = C_p / \left(\mu U_p\right). \tag{10}$$

An application with high r_a can have $r_a > \widehat{r}_p$ for several p and thus choose to supply them all, gaining access to different customers on each platform, i.e., multihome.[12] The multihoming, however, is not central to our point. Each platform could be supplied by different applications. As long as the $f_r(r)$ is the same on each platform, the analysis does not change.

An equilibrium is a point where $\chi_1(U_1) = U_1$. Stability analysis changes from the traditional version of the model in two ways. First, using Equation (10), as U_p grows, less attractive applications are supplied to the platform, so the margin $\widehat{r}_p$ falls.

$$\partial \widehat{r}_p / \partial U_p = -C_p / \left(\mu U_p^2\right).$$

As U_p increases, the absolute value of $\partial \widehat{r}_p / \partial U_p$ falls. This suggests more stability at larger U_p. Second, as $\widehat{r}_p$ falls, the aggregate contribution to user welfare will be larger or smaller depending on how many applications cross the threshold, i.e., on $f_r(\widehat{r})$:

$$\partial v_p / \partial \widehat{r}_p = -f_r(\widehat{r}_p)\widehat{r}_p.$$

Stability depends on the shape of the distribution of applications heterogeneity.

The new element of the positive feedback loop in this model is $V_p(U_p)$. We have:

$$V_p'(U_p) = \partial v_p(\widehat{r}_p(U_p)) / \partial U_p = -f_r(\widehat{r}_p)\widehat{r}_p \ \times \ -C_p / \left(\mu U_p^2\right) > 0. \tag{11}$$

For stability analysis we want to know when this derivative will be small, and, relatedly, when this derivative will be particularly small at a divided equilibrium with $U_1 = U_2$.

Those questions are easily answered if $f_r(r)$ is star-dominant. We define a star-dominant $f_r()$ as one for which $f_r(r)r$ is strictly increasing and convex. We use the label "star-dominant" because the contribution to user value on a platform, $v_p(\widehat{r}_p)$, defined in Equation (7), is larger for applications with larger r. This is illustrated in Figure 2a which shows the integral leading to v_p. The area under $f_r(r)r$ from $\widehat{r}$ up to 1 is $v(\widehat{r})$.

12. Our empirical model does not impose the assumption that r_a is the same across platforms. There is positive dependence: applications with high r on Android tend also to have high r on iOS.

Figure 2a: Star-Dominant Valuation Distribution

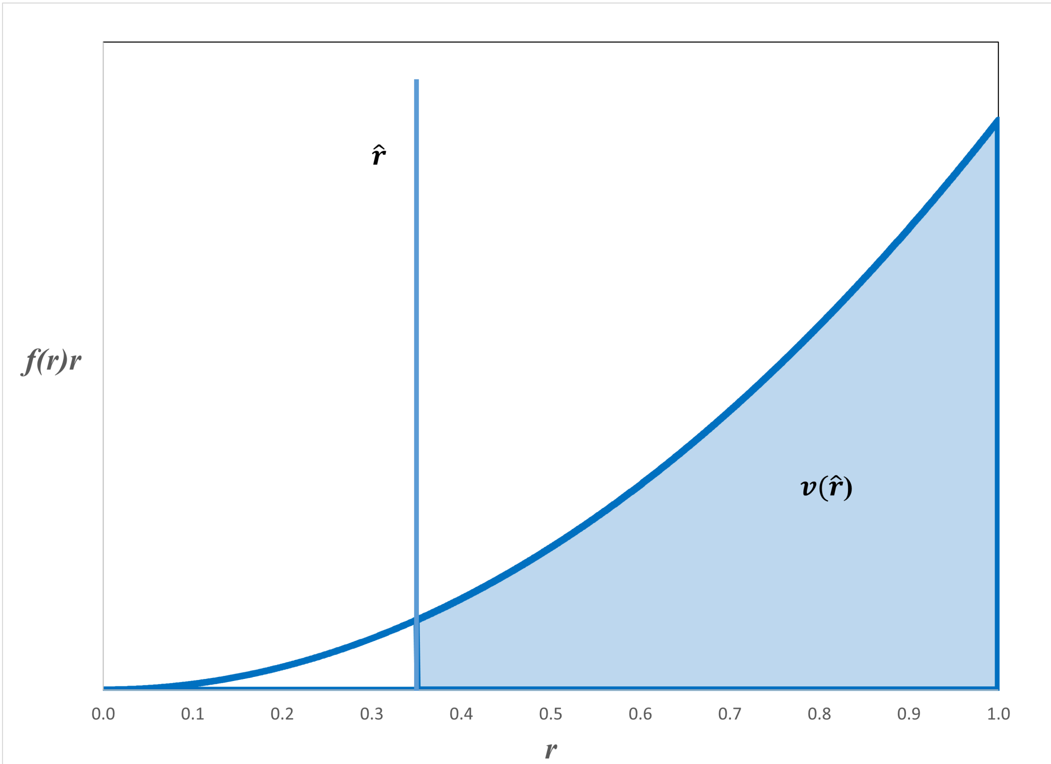
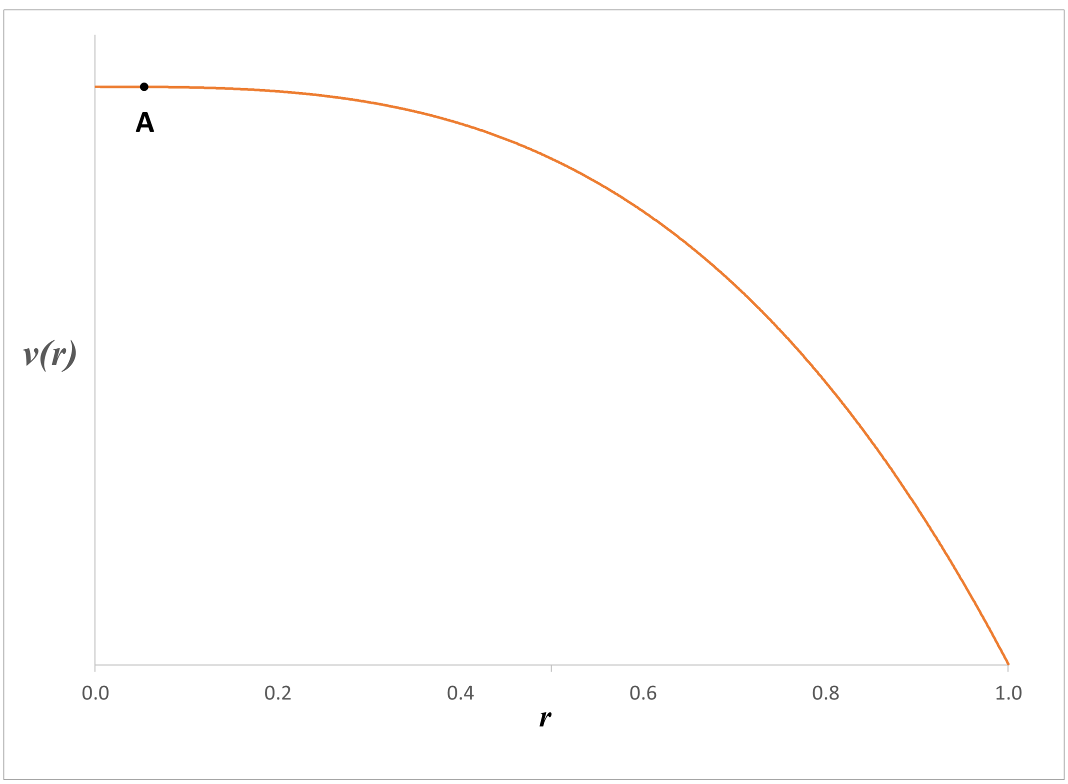


Figure 2b: Star-Dominant Valuation Distribution



Star-dominance leads to a concave shape for $v_p(\widehat{r}_p)$ as shown in Figure 2b. The function $v_p(\widehat{r}_p)$ has this shape because a small $\widehat{r}$ has a low level of $f(r) * r$. The marginal applications supplied to the platform at that point – the ones that would be just unprofitable if U_P were lower and thus $\widehat{r}_p$ were higher – do not make a large contribution to v_p. The shaded area marked as $v(\widehat{r})$ in Figure 2a contains many *inframarginal* applications which would not leave the platform if there were a small decline in U_p. If we were to consider a much higher $\widehat{r}$ than the one marked in Figure 2a, corresponding to a much lower U_p, this relationship would be reversed.

In Figure 2c, the orange curve shows $V_p(U_p)$, the supply of applications to platform p, corresponding to the same $f_r(r)$ shown in Figures 2a and 2b. Its shape reflects the same considerations. A large U_p leads to a small $\widehat{r}_p$: Under star-dominance this will be an inelastic supply point, like points marked A in Figures 2b and 2c. Those points are also ones where much of the value comes from inframarginal applications, which leads us to draw the average slope of $V(U)$ from $U_p = 0$ to point A as the blue line segment in Figure 2c. While $V'(U)$ is small at A, it must be larger at some smaller U: At smaller U, some of the attractive apps that are inframarginally supplied at A will be on the margin, and $V(U)$ will be steeper.

This inframarginal supply of the most attractive applications, and their large contribution to v_p, drives our results. A star-dominant $f_r(r)$ has two implications for the supply of applications to platforms, $V_p(U_p)$, that are central to stability analysis. A star-dominant $f_r()$ implies that $V_p'(U_p)$ is decreasing and convex in U_p. A star-dominant $f_r()$ also implies that $U_p * V_p'(U_p)$ is decreasing in U_p (see Appendix for demonstration). Star-dominance is clearly a powerful assumption.

As in the traditional version of the positive feedback model, the (in)stability index is the product of the slope of user platform demand (here $U(V)$) and of developer platform supply (here $V(U)$). We evaluate the (in)stability index under the assumption of symmetry of the economic fundamentals across platforms, so that $V_1(U) = V_2(U) = V(U)$:

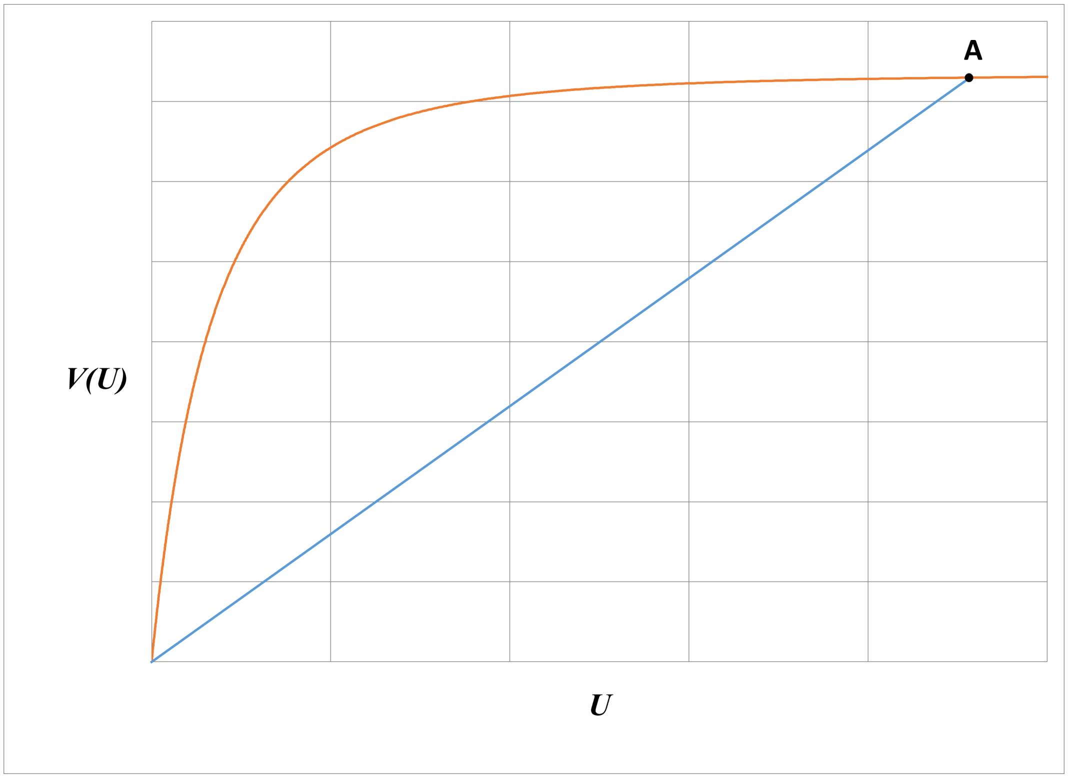
$$\chi_1'(U_1) = \gamma * f_u(\Delta_V) * U_M \Big[V'(U_1) + V'(U_2) \Big]. \tag{12}$$

The bracketed term gives the contribution of developer supply to the (in)stability index. Since star-dominance implies that $V'(U)$ is convex, we see that the contribution of applications supply to platforms to tipping is *at a local minimum* at divided equilibrium ($U_1 = U_2$). This is in contrast with the traditional version, where the contribution is at a local *maximum* at a divided equilibrium.

The implication is that the applications-heterogeneity variant can have a stable, divided equilibrium that is not unique.[13] The contribution of applications supply to platforms can be low at a divided equilibrium but

13. As in the traditional version, the applications-heterogeneity model can have multiple equilibria. Like the traditional version, the applications heterogeneity model will always have an equilibrium, will always have a stable equilibrium, and, under

Figure 2c: V(U) Associated with Star-Dominant $f_r(r)$



substantial if one platform has a low user installed base. In short, the equilibrium correspondence can have the form shown in Figure 3, with a stable divided equilibrium flanked by unstable equilibria with a dominant platform. Indeed, whenever the symmetric applications heterogeneity model has multiple equilibria, the divided equilibrium is stable.

Examining the geometry of Figure 3 in light of Equation (12) makes obvious the sufficient conditions for a stable, divided equilibrium which is not unique. In the figure, χ_1 has a slope less than 1 when the installed bases of the two platforms are similar, but it has a much larger slope if one of the platforms has a small installed base. Both of these conditions arise if star-dominance is sufficiently strong. First, star-dominance leads to a local minimum of the slope of supply, the term in square brackets in (12), at a divided equilibrium. Second, $\chi_1(U_1)$ is steeper when evaluated at $U_1 \neq U_M/2$, because increasing U_1 and decreasing U_2 by the same amount *increases* the term in square brackets in Equation (12) (see Appendix for demonstration). Quantitatively important star-dominance makes $V'(U)$ convex, i.e., $V'(U_1) + V'(U_M - U_1)$ is larger than $2V'(U_M/2)$. If $V'(U)$ is sufficiently convex, it will overcome the other elasticities and create the shape seen in the figure.

Whether this occurs in any particular market is an empirical question. User platform demand still

symmetry, will always have a divided equilibrium.

contributes a local maximum to tippyness at a divided equilibrium, just as in the traditional version of the model. In light of this, we report empirical stability results through a bound, the smallest elasticity of user platform demand with respect to v consistent with instability.

While the shape of χ_1 in Figure 3 is novel, it is not outlandish. We calculated it based on our model estimates of applications supply $V(U)$ and of $v(\widehat{r}(U))$, and on an assumed explosive response to v_p in user platform choice.[14] Figure 3 shows a stable divided equilibrium, even with the explosive response by users to available applications. However, this is not because the model rules out tippiness. The figure also shows two adjacent equilibria, one with mostly Android usage and more applications for Android, the other with mostly iPhone usage and more applications for iPhone. Critically, these divided equilibria are unstable. Each shows a powerful tendency for the smaller platform to tip out. Once again, the marginal developer to the smaller platform is a very valuable developer to users, so small changes in installed base have large impact on user value.

Finally, the comparative statics of stability at the divided equilibrium as market size (U_M) changes involves not only the supply of applications to platforms but the fact that increases in market size scale up U_p proportionately to U_M. On net, the (in)stability index is proportional to $U_M * 2V'(U_M/2)$; letting $U_m = U_M/2$, we can rewrite this as $U_m V'(U_m)$. Since this expression is decreasing in U_m under star-dominance, the (in)stability index at the divided equilibrium falls as market size increases.

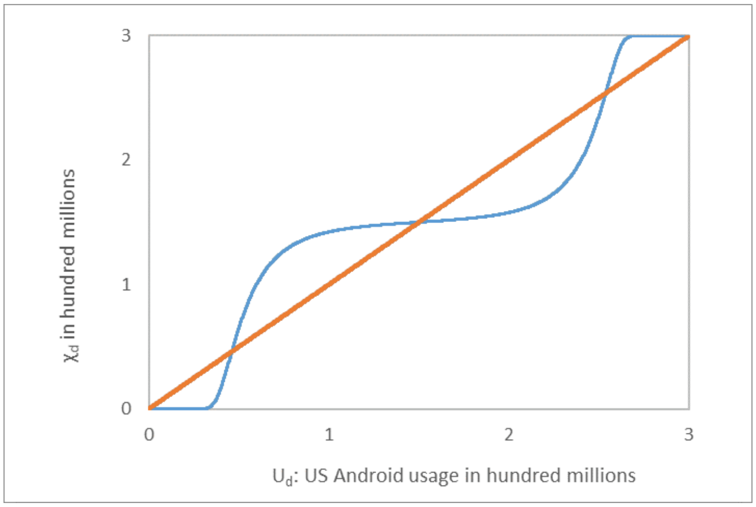
Indeed, for sufficiently large U_M, the divided equilibrium is stable. The relationship of stability to market size is the opposite with heterogeneity in app attractiveness than in the traditional version of the model.[15]

Users valuing platforms for their applications and applications valuing platforms for their users can generate multiple equilibria and tippiness. However, additional economic assumptions limiting applications heterogeneity are required to generate the tendency for divided equilibria to be unstable. The shape of that heterogeneity and its implications for stability can be studied empirically, the topic to which we now turn.

14. Specifically, Figure 3 is based on estimates from Table 2. User platform demand is a two-parameter logit calibrated to predict shares at the divided equilibrium and to have an elasticity of demand for iPhones with respect to v_{iOS} of 10. The divided equilibrium is not exactly symmetric since our estimates vary by platform, and there are other gaps between the empirical model and the theory. See Appendix VIII.C for a complete description.

15. Of course, it would be possible to get this result in the traditional version of the model by limiting positive feedback and positive network effects, perhaps by having more competition among applications on larger platforms (a source of negative feedback) or by adding diminishing returns to applications attractiveness so that the diminishing returns offset the social increasing returns of the platform. With applications heterogeneity, the result arises without removing positive feedback and positive externalities.

Figure 3: Multiple Equilibria with Applications Heterogeneity



IV Industry and Data

Today's mass market consumer smartphone industry began with the 2007 introduction of the iPhone.[16] The iPhone came with a more consumer-friendly design than any previous smartphone. Smartphones filled the market gap for a competent consumer computer and became the fastest growing, and soon the largest, development platform market ever. Although Google's Android was introduced 16 months after the iPhone, Google's open systems strategy allowed it to quickly catch up to Apple's user base and app supply. Platform shares were volatile for a period, but since early 2013 there has been little movement away from an approximately equally divided US platform market. Android's user installed base has been about 5/4 of iOS on smartphones (see Figure 4).

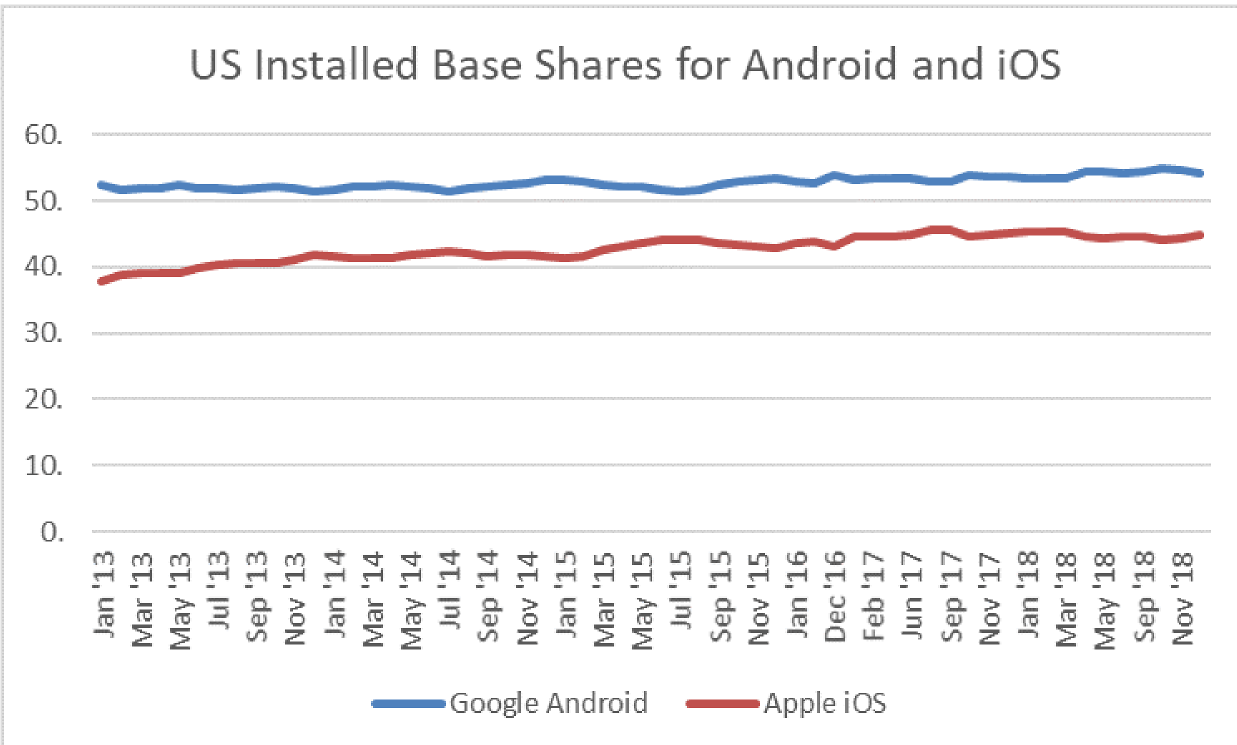
No comparable installed base dataset covers a wide variety of countries. It is clear, however, from data on the new-phone market and from data based on website access from smartphones, that the divided market structure of the US occurs only in a few countries.[17] The divided platform market equilibrium seen in the US is not the norm. Worldwide iOS share is around 14%, though demand for iOS with its expensive handsets is likely small in poor countries. Looking at rich countries, we see a wide range of equilibria: Japan and Denmark are about two-thirds iOS and Germany and France are under 30% iOS. Industry sources make reference to iPhone countries and Android phone countries.

Two smaller smartphone platforms tipped out of the US market. Blackberry had been dominant in

16. Before that, RIM dominated a much smaller market with its Blackberry devices for business communicators, and Nokia offered a "smart" phone that was not an important development platform. See discussion in Bresnahan and Greenstein (2014) about those firms' loss of dominance in the new consumer-oriented platform market.

17. New phones sales are difficult to convert into national installed base via a perpetual inventory method because there is a wide variety in the useful life of phones – most "burner" phones are Android, and there is a lively international market in used phones. Web access data, such as those from DeviceAtlas or StatCounter, somewhat overcount iOS because of iOS users' tendency to be richer and to use commercial services more. The figures quoted in text are from StatCounter.

Figure 4: Divided Platform Equilibrium in US



Source: Comscore "Subscriber Share held by smartphone operating systems in the Unted States." (https://www.statista.com/statistics/266572/market-share-held-by-smartphone-platforms-in-the-united-states). Shares sum to < 100%, as only top two platforms shown. The figure shows a three-month moving average stock of phones, labeled by the first of the three months; thus it is smoother than comparable figures based on new phone sales, which show a seasonal saw-tooth from the annual release cycle of new iPhone variants. Note that both Android and iOS are used in devices other than smartphones, such as tablets.

smartphones in an earlier, business-user era. The existing applications running on Blackberry phones were broadly irrelevant to consumers, and Blackberry's business user installed base was soon much smaller than the mass market consumers served by the iPhone. Blackberry, after difficult technical and management decisions, switched to a more consumer-friendly strategy, but found itself in a downward tip with too few users to attract apps and too few consumer-oriented apps to attract users.[18] Microsoft, a late but well-funded entrant into consumer-oriented mobile phone platforms, found itself with tipping forces pushing "Windows Mobile" out. On the developer side of the platform, Microsoft paid small bounties to each developer who submitted an app, and very large bounties, reportedly up to $100,000, to selected developers.[19] Unsurprisingly, the negative prices did not draw developers' best work. On the user side, Microsoft bought Nokia, an important smartphone platform firm in Finland, and switched its phones to Windows Mobile. With limited availability of quality applications[20] Windows Mobile tipped downward. Microsoft took a $7.6 billion charge, laid off more than 3,000 Finns, and then sold Nokia. While the Android/iOS duopoly appears stable, there are important tippyness elements in the US market. We cannot estimate app supply to these two smaller platforms, as industry sources do not find it worthwhile to collect suitable data on them. We shall, however, consider whether our model can explain their tipping out under the assumption that app supply to them and app heterogeneity on them resembled the larger two platforms.

We study the apps supplied by third party developers. A few apps are officially sponsored. They are the main software candidates for divided technical leadership, e.g., browser, mail, maps. We treat those as part of the fixed attractiveness of the platform to users. Exclusive contracts with outside developers are rare at the two largest platforms.[21] Central to any platform industry is all the technologies and services an application developer need *not* provide. For both the iOS and Android platforms, these include computer hardware (the phone) networking services (via cell networks and wi-fi) and system software running on the phone, on the web, and so on. Platform app stores also provide distribution services.

While all those technologies and services are available on both platforms, there are important differences in how they are structured and managed. Apple is vertically integrated, while Android smartphones come from a wide variety of sellers. Typically there are only a few iPhone models, and typically they are expensive.[22]

18. See Bresnahan and Greenstein (2014) for more complete analysis of this incident and for cites to industry sources.

19. See Bass and Satariano (2010). Microsoft also created programming frameworks that allowed applications to be shared between Windows PCs and mobile devices. The ease-of-porting strategy has worked well for Microsoft tablets but was ineffective for smartphones.

20. Industry observers noted this. William Stofega, IDC program director for mobile phones, was quoted as saying: "The quality of the apps in the store was inferior – not every one of them – but a lot compared to Apple or Android." Wheelwright (2017).

21. A few predictably popular games were exclusive to a platform for a short period of time at initial launch but not long-term. Also, both iOS and Android have used Yahoo apps to fill gaps before supplying officially sponsored apps (see Figure 5).

22. Bundling the smartphone with a cell subscription provides a source of financing for consumers. See Sinkinson (2020) for a very interesting strategic discussion of this practice in the context of the short-lived exclusive availability of the iPhone on AT&T.

Android phones vary widely in price and features. The large business advice literature for app developers, and our many dozens of interviews with developers, suggests that per-customer profits will be higher on iOS than on Android.

Developer costs vary between the platforms. Apple mandates app distribution only through its app store, with an approval process that imposes costs of consumer protection and security review. Apple restricts app access to many phone and operating system features. Comparatively permissive Google lets developers distribute through third party stores, and has a less-intrusive security policy.[23] On the other hand, the wide variety of Android phone screen shapes and sizes means that developers bear additional user-interface development costs on that platform. UI costs are typically a large fraction of the fixed costs of a mass market app, second only to marketing costs.[24] Our empirical model will measure, in the notation of (9), C_p/μ_p but we cannot measure the two elements separately. We have a strong conjecture that μ_p is higher for iOS but there are good reasons why C_p could go either way.

We measure the attractiveness of a mobile app on a platform by the fraction of the platform's users that demand the app. Apps are heterogeneous in their attractiveness to users for a variety of reasons. Some have powerful network effects themselves, like Facebook or Twitter. Others help consumers use an already-popular service, like United Airlines or Citibank. For our purposes, what is important is *that* a particular app is very attractive to consumers, not *why* it is.[25] If app prices or revenues were systematically observable, this measure could likely be improved. The wide variety of app "monetization" strategies, including earning the economic return entirely through selling complements to the app, render this impractical.[26]

23. Effective competition from Android led Apple to relax some of its developer restrictions. Similarly, Android has overcome some of its early shortcomings as a development platform for commercial apps, such as weak payment systems, and its security model has moved towards Apple's over time.

24. The largest costs of entry onto a platform are the marketing costs to "gain visibility", i.e., to make a new population of users on the platform aware of the app's existence. According to our discussions with industry participants (Li, Bresnahan and Yin (2016)), launch campaign costs average approximately $0.5 million. Entrepreneurial app developers tell us that they buy ads displayed in other app developers' apps and pay for "incentivized downloads" in an effort to gain visibility in a mass market. For this reason, our model allows costs to differ by platform and does not restrict the joint costs of multihoming to be less. By contrast, corporate apps can directly access their established firms' existing customers. We thus also permit costs to vary by developer type.

25. There are other industry structure and firm structure inquiries for which taking into account that an application itself has a positive feedback loop would likely be important. (1) The suppliers of both the Android platform and the iOS platform are vertically integrated into a number of applications, such as web browser, email client, and maps, with substantial network effects. It would be unwise to examine those decisions without considering the network effects as a central feature of the apps. These applications might be a source of divided technical leadership if left under the control of an outside firm (Bresnahan and Greenstein (1999)). Apple switched from primarily relying on Google Maps, a very successful product with large network effects, to create Apple Maps. Not taking that decision might have left control of a key complement to iOS in the hands of a direct platform competitor, one of the most problematic forms of divided technical leadership for strategic platform sponsorship. (2) Analysis of the decision by Google to enter mobile platform markets by buying Android not long after Apple founded the modern consumer-oriented mobile industry would be difficult without taking into account Google's already-existing businesses, such as search, which themselves have substantial network effects. Similarly, analysis of the decision by Apple to create the modern mobile platform industry would be difficult without taking into account Apple's already-existing businesses, such as its online music store.

26. See Bresnahan, Davis and Yin (2015) and Miric, Boudreau and Jeppesen (2019) for discussion of and statistics on monetization strategies. Some apps have comparatively price-like monetization strategies, such as an initial price or a recurring subscription price. These, however, are not as common as other forms, such as advertising-supported apps, apps with delayed pricing (either "freemium" or for enhancements), and corporate apps that support consumer product and services companies in their main lines of business outside of mobile (e.g., an airline) which typically do not monetize directly through the app.

In the US platform market, there are millions of economically unimportant applications on each platform. We build our empirical analysis around a commercial data set that reports information on applications used by at least 0.0012 of users on either iOS or Android phones in a large sample of users. Specifically, we use the January 2013 Mobile Metrix dataset from Comscore.[27] Comscore has two samples, one of 5,000 adult Android smartphone users in the US and a parallel sample of 5,000 iPhone users. Comscore only reports data on an app on a platform if there are more than 5 users in their sample for that platform.[28] Finally, we keep only applications that come from "independent software vendors," as it is the supply behavior of these developers and the demand for their applications that lie at the heart of our economic enquiry.[29] This yields our final sample of $1,044$ apps. We will address the sample selection issues shortly.

First, based only on the Comscore data, we define four variables for each app. We use the * notation to denote that a variable comes from Comscore and $p = d$ for Android and $p = i$ for iOS. S^*_{pa} is a dummy variable for the event "app a is observed on platform p in the Comscore data". At least one of S^*_{da} or S^*_{ia} is 1 for every sample app. For each platform, Comscore reports a projection of the fraction of the US population who used the app during the month. This fraction, denoted r^*_{pa}, is called "reach" in the industry. Obviously, r^*_{pa} is truncated from below at 0.0012 for all apps that are actually supplied to platform p, so both S^* and r^* are selected by Comscore's rules.

To improve our models of selection, truncation, and app supply to platforms, we go outside Comscore and define S_{pa} as a dummy for whether the app was in fact supplied to the platform. For each sample app that appears in the Comscore sample on only one platform, we undertook an extensive search to determine whether the app was also supplied to the other platform.[30] Obviously, $S_{pa} \geq S^*_{pa}$ since an app can be available on a platform but not used by more than 5 people in the Comscore sample.

We employ r^*_{pa} as our index of app a's attractiveness on platform p.[31] The dependent variables in our

27. This dataset is available for subscription at academic rates. We will provide our programs for processing it to anyone seeking to replicate this paper, but you will need to buy your own copy of the underlying data.

28. Comscore also reports a few apps with less than this level of usage if a client has requested tracking. We drop these. Modeling those requests in order to gain a few data points seems likely to lower, not raise, statistical reliability.

29. That is, we exclude apps produced by Apple, Google, carriers (e.g., Verizon), and OEMs (e.g., Samsung). These apps are often pre-loaded and thus may not reflect user demand – certainly not user downloads and often not user usage of the app. Carrier and OEM apps are not very important in total demand. Apple and Google, as we noted above, each provide a set of core applications that we think of as part of the platform itself, not the indirect network effects positive feedback loop.

30. Our main sources were the two platforms' app stores, developers' websites, app data sources Distimo and AppAnnie, and Crunchbase. For apps that appear on only one platform in Comscore, we first looked in AppAnnie and Distimo. If we can find the app, we are finished, since those sources report when and if the app was first available on the other platform. Linking to those sources is not always possible, since there are no developer nor app unique identifiers common across the app stores, much less to AppAnnie, Distimo, or Comscore. Our next source was the the developer's website, typically listed on the app store. That often led us to App Annie, or to a direct statement of whether or not the developer supplies to the other platform. For developers with an uninformative website or no link to it on the primary platform, we search in Crunchbase, which frequently lists available products by platform for firms listed there. Sometimes, apps with the same name on different platforms have links to the same developer website, in which case we treat them as the same app. If none of this information is dispositive, we verify whether it is a version of the same app as on the first platform by comparing screenshots of the apps.

31. The other candidate variable, total time spent by users in the application, appears to be badly measured and varies remarkably little conditional on r^*_{pa}. A plot of the joint distribution of reach and time appears in Bresnahan, Davis and Yin (2015). Most importantly, the distributions of both metrics are similar and both appear to be star-dominant.

Table 1: Descriptive Statistics (1,044 apps)

Variable	**Mean**	Variable	**Mean**
S_{ia}	0.765	S^*_{ia}	0.574
S_{da}	0.820	S^*_{da}	0.657
S_{ba}	0.647	S^*_{ba}	0.231
Mobile Era (O)	0.420		
Online Era (L)	0.290		
Offline Era (F)	0.290		
Publicly Traded (T)	0.300		
Game (G)	0.313		
Variable	**Mean (St Dev)**		
r^*_{ia}	0.021 (0.060)		
r^*_{da}	0.018 (0.050)		

S_{ia}, S_{da}, S_{ba} are indicator variables for whether the app was supplied to iOS, Android, or both. S^*_{ia}, S^*_{da}, S^*_{ba} are indicator variables for whether the app was observed in Comscore on iOS, Android, or both. r^*_{ia} and r^*_{da} are the reach of apps observed in Comscore on each platform. *Mobile Era, Online Era, Offline Era, Publicly Traded,* and *Game* are indicator variables for characteristics of the developer or app.

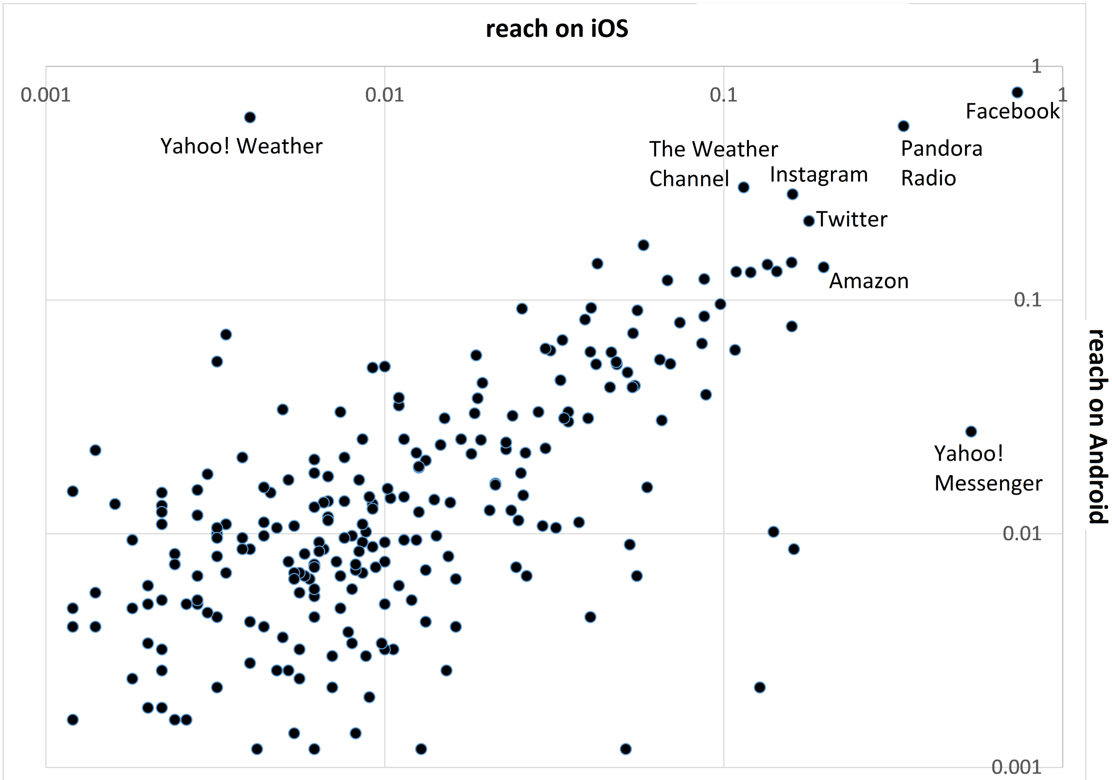
model are r^*_{ap}, S^*_{ap}, and S_{ap}.

Our regressors X are characteristics of app a or its developer which we will employ as type regressors, i.e., observable heterogeneity measures. The first is a feature of the app itself: *Game* (abbreviated G) indicates whether the app is in the game category.[32] We also use measures about the firm that supplies the app. We designate three mutually exclusive firm types based on the firm's technological era: offline, online, and mobile. If the firm was founded as a developer of mobile apps (e.g., Rovio), *Mobile Era* (abbreviated O) equals 1. *Offline Era* (abbreviated F) equals 1 if the developer had an offline business before having an online business or mobile app (e.g., Delta Airlines, CVS pharmacy). Finally, *Online Era* (abbreviated L) equals 1 if the firm has an online business, and was, at the time of its founding, an online-only firm (e.g., Facebook). Obviously $O + L + F = 1$. We also define *Publicly Traded* (abbreviated T) $= 1$ if the developer is a publicly traded firm. Descriptive statistics are reported in Table 1.

We show descriptive statistics in Table 1 and Figures 5, 6, and 7. The first three lines of the table show application supply to platforms as S_{pa} – the application is available on the platform's app store – next to S^*_{pa} – the app appears in Comscore's sample for the platform more than 5 times. An app is about 20

32. Other app category variables are available from Comscore, from the platforms' app stores, or from other commercial sources. However, the game/non-game distinction is the only categorization which is measured reliably. See discussion in Davis, Muzyrya and Yin (2014).

Figure 5: Joint distribution of r^* on Android and iOS for $S_b^* = 1$ (logscale)



percentage points more likely to appear on the iTunes app store than to be observed in Comscore's iOS sample (0.77 vs 0.57) and about 16 percentage points more likely on the Android side. We also constructed the multihoming or "both" supply row. Multihoming is common in our sample, but not universal, as about 2/3 of apps multihome. Far fewer apps multihome and achieve enough success on both platforms to appear in both Comscore samples, about 23%. Developers tell us that some apps simply fail to be discovered by the customers on a platform and end up with very low demand. We model this, and also model dependence in user app demand across platforms for the same app. Looking at Figure 5, the estimated degree of dependence will likely be high. In fact, $\text{corr}(r_{ia}^*, r_{da}^* \,|\, S_{ba}^* = 1) = 0.691$.

The figure also shows that the minimum of r_{pa}^* is (by construction of the sample) 0.0012 and the maximum is near 1. The means are both around 0.02, meaning that star applications are less frequent than mundane applications, as one would expect.

Figures 6 and 7 graph an empirical version of $v(\hat{r})$ (Equation (7)) for each platform. We take the set of observed r_{pa}^* and calculate the step function $v_p^*(\hat{r}_p) \equiv \sum_{r_{pa}^* > \hat{r}_p} r_{pa}^*$. These emplrical figures clearly have the star-dominant shape shown by the orange curve in (theoretical) Figure 2b. Any quantitative results will need to wait for model estimates. Howver, these simple descriptives suggested by the theory already demonstrate that application heterogeneity is a first significant digit force and that the shape of the distribution of app

Figure 6: Empirical v for iOS $v_i^*(\hat{r}_i)$

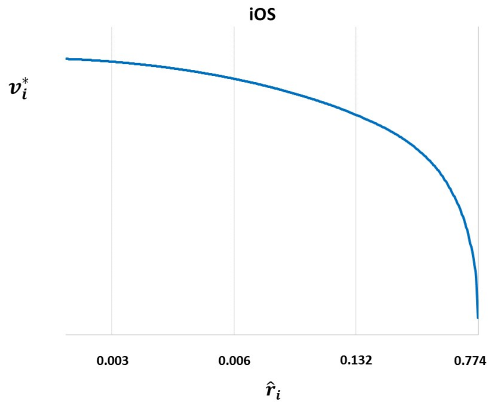
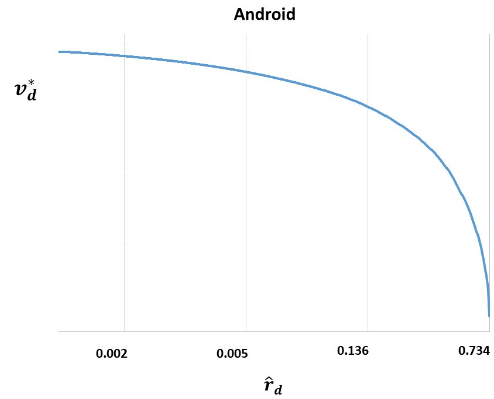


Figure 7: Empirical v for Android $v_d^*(\hat{r}_d)$



attractiveness is star-dominant. These figures show the information in the data that underlies our main economic conclusions.

Finally, the similarity of Figures 6 and 7 to one another, the visible symmetry of Figure 5, and the tendency of all the iOS and Android data in Table 1 to be similar to one another suggest that applications supply and demand are approximately symmetric across platforms in the divided US platform market. While there are somewhat more Android than iOS apps, there are not proportionately as many more as there are US Android users than iOS users.

The distribution of all the variables, including the Xs, in Table 1 reflect selection on the criterion that the app is economically important. The bulk of the millions of apps that are on the app stores or even of the tens of thousands of apps that have been on a top 500 list are from new entrepreneurial firms, but our sample includes only 42% Mobile Era firms (new) and 30% Publicly Traded firms (not newly entrepreneurial) as the supplier. Similarly, about half of the apps on the top free list are games, but only 31% of our sample are games. We shall examine whether this tendency for certain kinds of apps to be more valuable economically is still visible after we account for selection and see how selection impacts the inclusion of apps of different types in our estimation sample.

V Developer Supply to Platforms and User App Demand

In this section, we specify an econometric model of user app demand and developer supply with applications heterogeneity. It has the basic structure of the model in section III.B, above, but also includes a number of elements related to our industry. We seek to estimate the supply function of apps to a platform and the demand distribution of apps' attractiveness to users.

Figure 8: Timeline of Empirical Model

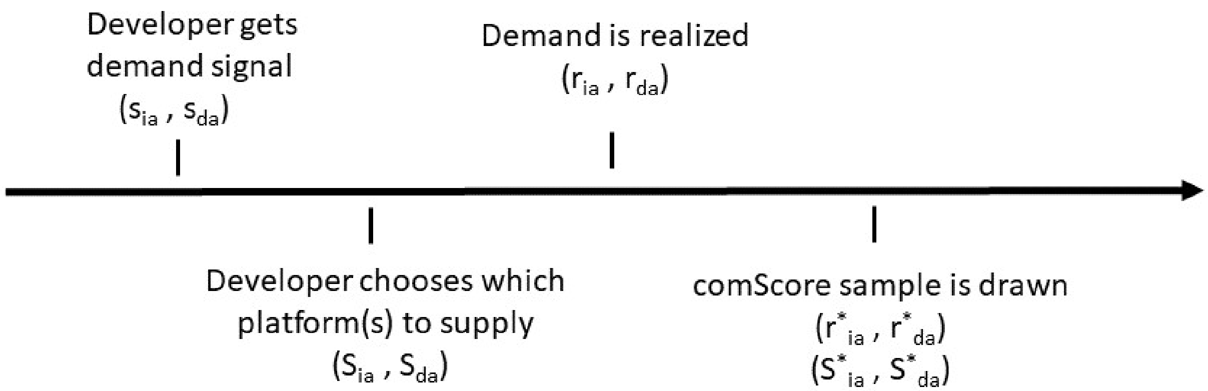


Figure 8 shows the model timeline. Developers may have incomplete knowledge of how well their app will be received by the consumers on a given platform. Some applications fail to gain much visibility with consumers on a platform. We treat these together in the two events above the timeline. A developer first gets a signal, $s_a = (s_{ia}, s_{da})$ of the app's demand on each platform. If the app is supplied to a platform p, it either gains visibility with a consumer or it does not, and the resulting reach is r_{pa}. The random variable r_{pa} has support $(0, 1)$, so we will model both reach and signals of reach as having *beta* distributions. We first model the distribution of the signal, then demand conditional on the signal. In our simplified theoretical model, r_a is the same on all platforms. In our empirical model, the developer gets the same signal of profitability (and thus has the same entry behavior) with probability ω_X. Let X_a be observable features of app a and of the firm supplying the app. We frequently write a deep parameter θ as depending on X_a and denote that by θ_X. If the parameter also varies across platforms, we write θ_{Xp}. Finally, we use the notation θ to mean the vector of all θ_X or of all θ_{Xp}.

We denote the common, cross-platform signal as q_b ("b" for both) and assume that it has a *beta* distribution. To permit many economic and econometric calculations in closed form within the model, we assume that the signal is a mixture of *beta* distributions.[33] Symmetry across platforms is not assumed but the distribution of q_{ba} is restricted to be an average of the two platform distributions

$$q_{ia} \sim beta(\alpha_{Xi}, \beta_{Xi}), \quad q_{da} \sim beta(\alpha_{Xd}, \beta_{Xd}),$$

$$q_{ba} \sim beta((\alpha_{Xi} + \alpha_{Xd})/2, (\beta_{Xi} + \beta_{Xd})/2).$$

33. There are several other ways to model dependent *beta* distributions. The Sarmonov method fits very badly in our application, since it limits the correlation to be near zero. Another method is to build up the distributions from ratios of *gamma* distributions. This, however, does not lead to both marginal and conditional *beta* distributions and thus would leave a number of the economic as well as econometric calculations below much more difficult.

We model the developer's signal draw as

$$s_a = \begin{cases} (q_{ia}, q_{da}) \text{ with probability } \omega_X, \\ (q_{ba}, q_{ba}) \text{ otherwise.} \end{cases} \tag{13}$$

To specify the relationship between the signal of reach and reach we introduce λ, the probability that all users learn about the app. If the app is not visible to users, then realized demand for the app, $\overline{r}_{pa}$, is drawn as a new random variable. It is a "shrunk" version of reach:

$$\overline{r}_p \sim beta(\delta_{Xp} * \alpha_{Xp}, \beta_{Xp}), \tag{14}$$

with $0 < \delta_{Xp} < 1$ parameters to be estimated. Then

$$r_{pa} = \begin{cases} s_{pa} \text{ with probability } \lambda_X, \\ \overline{r}_{pa} \text{ otherwise.} \end{cases} \tag{15}$$

Using (15), we can easily calculate $f(r_a \,|\, s_a, X, \lambda, \delta, \alpha, \beta))$. Using (15) together with (13), we can easily calculate the joint $f(s_a, r_a \,|\, X, \lambda, \delta, \alpha, \beta, \omega)$.

While our demand distribution is parametric and built of *beta* distributions, it is not highly restrictive. First, our model of the size distribution of app demand involves two mixtures, with parameters ω and λ, of *beta* draws. We do not restrict applications attractiveness to follow a β distribution. Second, there is observable heterogenity through the dependence on X. Further, our empirical model is more general than our theoretical model in that the gap between developer signals of demand and realized demand means that supply and demand at the app level are closely related but not identical.

Our model treats developer supply to platforms like entry into markets, and thus shares elements with models reviewed in Berry and Reiss (2007). If app a is published on platform p, it earns $\pi_{pa} = \mu_{pa} \times U_p \times r_{pa} - C_{pa}$, where μ_{pa} is the marginal profit per customer of the app to the developer, U_p is the number of users on platform p, r_{pa} is the fraction of users who demand app a on platform p (reach), and C_{pa} is the fixed costs of supplying app a on platform p. In our timeline, the realization of r_a is not known to the developer at the time of the entry decision, so conceptually the condition for supplying a platform to be profitable is

$$\mu_{pa} \times U_p \times \mathbb{E}[\, r_{pa} \,|\, s_a \,] \geq C_{pa}. \tag{16}$$

The supply condition is, of course, an entry threshold, but $\widehat{r}_{Xp}$ is now the smallest value of the *signal* for which (16) is satisfied. Given our parametric assumptions, we can solve for that in closed form in two steps.

First, letting θ be an abbreviation for all parameters,

$$\mathbb{E}[\, r_{pa} \,|\, s_a, X_a, \theta \,] = \lambda_{Xp} s_{pa} \;+\; (1 - \lambda_{Xp})\mathbb{E}[\, \overline{r}_{pa} \,|\, X_a, \theta \,].$$

We define supply parameters, κ_{Xp}, as the ratio of fixed costs to variable profit, $\kappa_{Xp} = C_{Xp}/\mu_{Xp}$. (We cannot in principle separately identify μ and C.) Now $\widehat{r}_{Xp}$ solves

$$\lambda_{Xp}\widehat{r}_{Xp} \;+\; (1 - \lambda_{Xp})\mathbb{E}[\, \overline{r}_{Xp} \,|\, X, \theta \,] \;= \kappa_{Xp}/U_p. \tag{17}$$

This leads us to normalize κ for estimation as κ_{Xp}/U_p, putting it in the same units as developer signals and reach. Thus an app is supplied to a platform, i.e., $S_{pa} = 1$, if it has $s_{pa} > \widehat{r}_{Xp}$ where

$$\widehat{r}_{Xp} = \kappa_{Xp}/\lambda_{Xp}/U_p - (1 - \lambda_{Xp})/\lambda_{Xp} \times \mathbb{E}[\, \overline{r}_{Xp} \,|\, X, \theta \,]. \tag{18}$$

This lets us make the first of several steps in calculating the likelihood. The supply to each platform of apps of observable type X is the set of signals above the value for $\widehat{r}_{Xp}$ given by (18). The distribution function for those signals comes from (13). We calculate $Pr(S \,|\, X, \theta)$, the probability of any of the four possible values of S_a, as a function of X_a and parameters. This is not yet corrected for selection (the event $S = (0, 0)$ cannot occur).

Above, we showed the calculation of $f(r_a \,|\, s_a, X, \lambda, \delta, \alpha, \beta)$. Since S_a is just a coarsening of s_a, this makes calculating $f(r_a \,|\, S_a, X, \lambda, \delta, \alpha, \beta)$ simple in principle, and our parametric assumptions mean that we can make this calculation in closed form. Next, we deal with the truncation problem that, conditional on $S_p = 1$, we will only observe r^*_p if it is at least 6/5000. Then we deal with the selection problem that we only observe an app at all if it satisfies that condition on at least one platform.

Conditional on $S_{pa} = 1$, the probability that an app is observed in the Comscore sample and the observed sampling distribution of r^*_{pa} follow from the distribution of r_p which is, conditional on $S_{pa} = 1$, a *beta* mixture. Let g_p be the number of platform p users that have the app in the Comscore sample. The distribution of g_p conditional on r_p is binomial and involves no new parameters as the sample size of 5000 is known; unconditional on r_p, g_p has a $beta - binomial$ mixture. The app is in the Comscore sample if $g_p \geq 6$. Thus $Pr(S^*_p = 1 \,|\, S_p = 1, X, \theta) = Pr(g_p \geq 6 \,|\, S_p = 1, X, \theta)$.

Next, we calculate $f(r^*_p \,|\, S^*_p = 1, S_p = 1, X, \theta)$, noting that the realization of r^*_p is always a value of the form $k/5000$, where k is an integer, using $Pr(g_p = k \,|\, g_p \geq 6, X_a, \theta)$ for any k from 6 to 5000.

We can now write the joint distribution of r^*, S^*, and S given X and parameters θ, denoted $f_Y(S, S^*, r^* \,|\, X, \theta) \;=\; Pr(S \,|\, X, \theta) * Pr(S^* \,|\, S, X, \theta) * f(r^* \,|\, S, X, \theta)$. We do not observe an app unless it meets Comscore's sampling criterion on at least one platform, i.e., $S^*_{ia} + S^*_{da} > 0$. The structure of the joint distribution of r^*, S^*, and S and our parametric assumptions permit us to calculate the probability of this event, denoted $\Pr(S^*_{ia} + S^*_{da} > 0 \,|X_a, \theta)$ in closed form. We correct for sample selection by dividing by this probability and maximizing the conditional likelihood:

$$L_C(S, S^*, r^* \,|\, X, \theta) = \sum_a \log(\frac{f_Y(S_a, S^*_a, r^*_a \,|\, X_a, \theta)}{\Pr(S^*_{ia} + S^*_{da} > 0 \,|\, X_a, \theta)}).$$

The selection correction, as we noted above, solves a long-standing "potential entrants" problem. Our list of potential suppliers is the observed suppliers on the other platform. It is endogenous, as is the list of potential entrants made up of observed entrants in adjacent markets in other studies. Our model adds no additional "selection" parameters to be estimated because we build economic and econometric models of entry into, and being observed in, either or both markets. This creates not only a model of supply but also a model of selection, as entry into at least one of the markets is how a firm qualifies for the list of potential entrants into the other.

VI Results

We first estimated our model without restrictions. That means letting all the deep parameters vary with X and, where possible, with p. That specification led to many poorly-estimated parameters. Thus we report the more restricted specification where the free parameters are those listed in Table 2.[34]

VI.A App Demand on iOS and on Android

We begin with the demand estimates, reported at the top of Table 2. We write α_{Xp} as a regression on the supply firm characteristics F, O, and T and the app characteristic G. Thus the baseline constant α_p applies to the values Online Era (L), Non-Game (Y), and Privately Held (not publicly traded) (H).

The first row of Table 2 indicates that $\alpha_i \approx \alpha_d$ for baseline apps/firms. Equality across platforms is less clear in the point estimates of the rest of the α parameters, but we cannot reject the hypotheses that $\alpha_d = \alpha_i$, either when we test one row at a time or for the entire α_p vector. There appears to be little statistical difference between app demand on the two platforms. Now turning to the question of α varying with X, we reject the hypothesis that X does not matter for α statistically. The larger effects are for Mobile Era (O) firms (negative on both platforms, significantly so on iOS), Publicly Traded (T) firms (positive on both, significant on iOS) and Game apps (negative on both, significant on Android). We cannot reject the hypothesis that β and ω do not vary with X or p and have imposed this restriction in the results shown. The

34. We shut down variation over X in the dependency parameter ω, and all but one X coefficient in the uncertainty parameters λ. We set $\delta = 0.02$, consistent with findings in Li, Bresnahan and Yin (2016) regarding the difficulty of getting visibility on a top list in the app stores, which uses time series data on the apps' progress in app store top lists. While in this specification only the α_{Xp} vary across observable type X and platform p, we also have a more richly parameterized specification in which β_{Xp} also vary. Little differs in that specification, so we do not present those results here. κ/U varies with p but not X.

Table 2: Parameter Estimates

	Estimate	Standard Error		Estimate	Standard Error
α_d Constant	0.3124*	(0.1035)*	α_i Constant	0.3003**	(0.0984)
α_d Offline Era (F)	-0.0083	(0.0710)	α_i Offline Era (F)	-0.0000	(0.0795)
α_d Mobile Era (O)	-0.1195	(0.1175)	α_i Mobile Era (O)	-0.2005*	(0.1088)
α_d Game (G)	-0.0781**	(0.0328)	α_i Game (G)	-0.0224	(0.0252)
α_d Publ. Traded (T)	0.0670	(0.0550)	α_i Publ. Traded (T)	0.1485**	(0.0672)
β	22.5658**	(0.4536)			
ω	0.3482**	(0.0631)			
κ_d/U_d	0.0011**	(0.0004)	κ_i/U_i	0.0011**	(0.0004)
λ	0.6480**	(0.0728)	λ (O)	-0.0983	(0.1158)

Note: Bootstrapped standard errors are presented (250 draws). **significant at 5% *significant at 10%. The parameters β, ω, and λ do not vary by platform. The parameters α and κ/U do vary by platform.

estimate of ω of a little over a third suggests substantial but not overwhelming dependence across platforms in the unobserved portion of app demand. The estimate of β is quite large, and the regressors in α are all dummy variables, so we can immediately see that $\beta \gg \alpha$ for any X. This implies the unsurprising result that the mean predicted demands are small.

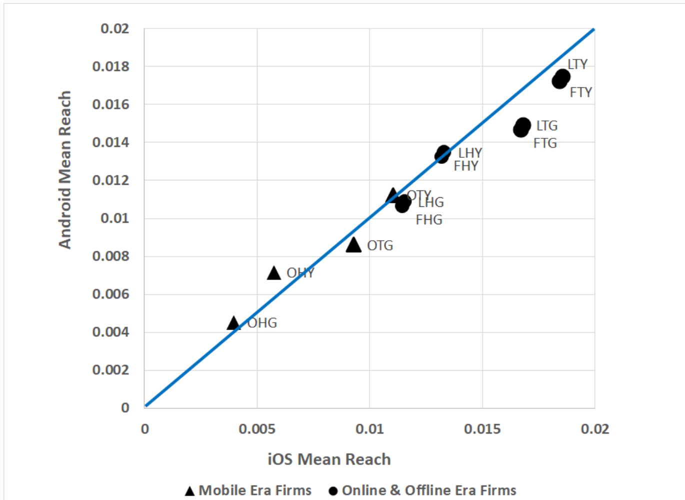
We can see the structure of observed heterogeneity in app demand across firm and app characteristics and across platforms in Figure 9. For each of the 12 unique values of X we plot the point estimate of the mean of expected reach on iOS and on Android if consumers learned about all apps.[35] Each point is labeled by its X values; for example, the leftmost point is OHG, i.e., Game apps from Mobile Era, Privately Held firms. The notation for all the other points is explained in the table footnote.

Our estimates reveal different roles for observable variation across apps in (fully informed) demand as X and p vary. First, as shown in Figure 9, the mean of fully-informed demand conditional on X does not vary much with p. In light of the symmetry across p of the joint distribution of observed r^* seen in Figure 5, this is unsurprising.

However, the figure also shows that the mean of fully-informed demand does vary with X. This variation, however, is not a large portion of the total variation in fully informed demand across apps. We calculate two model-based quanta. First, the population-weighted variance of mean demand as X varies is $var_X(E[Y|X,p])$. This is the population weighted variance of the means in Figure 9. Second, we calculate the variance of an individual app's demand, if consumers were fully informed, around that mean, $E_X[var(Y|X,p)]$. We calculate this variance for each X, and then take the weighted average over X. The

35. That is, for iOS it calculates $\omega * \alpha_{Xi}/(\alpha_{Xi}+\beta)+(1-\omega)(\alpha_{Xi}+\alpha_{Xd})/(\alpha_{Xi}+\beta+\alpha_{Xd}+\beta)$ and symmetrically for Android. These follow from our assumption that developers' signals are the reach of their app if users could learn about the apps without search. In another notation, the figure shows $(E[s_i|X], E[s_d|X])$.

Figure 9: Mean Reach by Platform and X if all apps were visible to users



Points are labeled by their X in the notation ABC, where C=G (Game) or Y (Non-Game), B=T (Publicly Traded) or H (Privately Held) and A=O (Mobile era), L (Online Era), or F (Offline Era). The L and F points are marked with a different symbol than are the O points.

first of these quanta is the observable variation in fully informed demand, the second, the unobservable variation. The unobserved variation is 13x as large for iOS, 18x as large for Android. In light of the very wide spread in realized app demand seen in Figure 5, this too is unsurprising.

There is a clear, unexpected, pattern in the variation of mean demand across X. To see it, we have used different symbols in the figure for mobile era firms vs. firms founded in earlier eras. The established firms – both ones from the older technological eras and publicly traded ones – tend to have higher demand. Entrepreneurial firms from the mobile era tend to have lower demand. This contradicts an expectation held by many industry observers that mobile app development would be an entirely new, entrepreneurial industry. It certainly is very entrepreneurial if you count firms, but not if you count market success.

That pattern in demand if consumers were fully informed about all apps is reinforced by the point estimates of λ, the probability that consumers are fully informed. The point estimates for λ are approximately two-thirds, though smaller for Mobile Era apps (λ (O) < 0). Though imprecisely estimated, this suggests that Mobile Era apps have a lower probability of being visible and achieving their fully-informed consumer reach. In short, a version of Figure 9 displaying the point estimates of the means of realized reach r_{Xp} using equation (15) would reveal an even greater relative demand disadvantage for Mobile Era firms' apps.

While we can reject the model of our theory section in which apps have the same attractiveness to users of both platforms, demand for apps is quantitatively similar on iOS and Android. When we turn to the analysis of stability based on our estimates, we will use these quantitatively similar but not identical demand estimates.

VI.B Supply for iOS vs. Android.

The supply parameters (κ) are presented in Table 2. We estimate them as κ_p/U_p since those are the units of reach and developer's signal. The table shows that estimated $\kappa_i/U_i = \kappa_d/U_d$ to three significant digits (they also cannot be distinguished statistically). This result has two economic implications.

First, these estimates imply $\mu_d/C_d = 0.75\mu_i/C_i$.[36] That is, [per customer return]/[fixed costs] is 75% as large on Android as on iOS. This is consistent with the developer opinion we discussed above, in which per-customer profits are higher on iOS, since iOS users tend to be richer. Second, the higher μ_i/C_i is just offset by the lower U_i and therefore $\mu_p/C_p/U_p$ is almost exactly equal on the two platforms. This leads to very nearly symmetric supply behavior between the two platforms.[37] The source of this symmetry in the data is not surprising. First, the demand estimates are very close to symmetric across platforms. Second, as we saw in Table 1, approximately the same fraction of sample apps were written for iOS (77 percent) as for Android (82 percent).

Our sample-selection calculation varies with X, since both α and λ do. Apps with lower realized demand are more likely to fall below the threshold for inclusion in the sample. For example, $\Pr(S^*_{ia} + S^*_{da} > 0 \mid X, \theta)$ is about half as large for Mobile Era applications as for others. See details in Appendix Table 4.

Overall, the distribution of applications heterogeneity, the degree of developer uncertainty about applications success, and developer profitability post-entry (adjusted for slightly different installed bases) are similar across the two platforms. While we do not estimate a user platform demand equation, the 5/4 split in favor of Android is consistent with somewhat higher prices for iPhones and with a slightly lower v, total value of applications, on iOS (calculated in Table 3). The divided US equilibrium is close enough to the theoretical construct of symmetry to allow investigation of the question of stablity.

VI.C Platform Market Equilibrium Stability

We now examine the equilibrium (in)stability index based on our empirical estimates, providing a bound for stability at the historical US duopoly, in a smaller market, and at an asymmetric equilibrium with one smaller platform.

An equilibrium is a fixed point of the function of U to itself, $\chi(U) = D(v(\widehat{r}(U)))$, and its Jacobian is given by $J_D J_v J_{\widehat{r}}(U)$. We have estimated $J_v(v \mid X, \theta)$ and $J_{\widehat{r}}(U \mid X, \theta)$ and will calculate bounds on J_D.

One change from the theoretical model is that the estimates of application heterogeneity, the quality of

36. Since $U_i/U_d = 0.39/0.52$ in January-March 2013, the period including our estimation month.

37. Since the thresholds to supply each platform depend on expected demand and on λ as well as on κ_p/U_p (see Equation (18)) there are small differences in $\widehat{r}_{Xp}$ across p and larger differences across X. The absolute value of $\widehat{r}_{Xd} - \widehat{r}_{Xi}$ is less than 0.0015 for 10 out of 12 values of X. The largest difference arises for Publicly Traded, Offline Era firms supplying a Game app, where $\widehat{r}_{Xd} = 0.0090$ and, $\widehat{r}_{Xi} = 0.0115$.

developer signals, and supply costs can vary with X and p. As we show in Appendix VIII.D.1, $J_{\widehat{r}}(U)$ is a $P*dim(X)\times P$ matrix where P is the number of platforms under consideration and $dim(X)$ is the number of unique values of X with typical element: $\partial\widehat{r}_{Xp}/\partial U_p = -C_{Xp}/(\mu_{Xp}\ \lambda_{Xp}\ U_p^2)$. The same changes are relevant to our empirical J_v. As we show in appendix VIII.D.2, the typical element of the $P \times P * dim(X)$ matrix J_v is

$$\frac{\partial v_p}{\partial \widehat{r}_{Xp}} = N_X\left[-\lambda_{Xp}\widehat{r}_{Xp}f_{Xp}(\widehat{r}) + (1-\lambda_{Xp})E[\overline{r}_{Xp}]f_{Xp}(\widehat{r})\right], \tag{19}$$

where N_X is the population size of potential developers of apps of type X and $f_{Xp}()$ is the density function of the signal seen by the developer.[38] We note that the $P \times P$ matrix $J_vJ_{\widehat{r}}(U)$ depends both on estimated parameters and on the point at which we are evaluating it, U.

Without an estimate of $D(v)$, we proceed to bound it. We assume it is logit and that γ is the coefficient of v_p in the indirect utility of platform p. Our bounds are calculated by increasing γ until the relevant equilibrium is just unstable. We report the size of γ in terms of the elasticity of demand for the iOS platform with respect to v_i.

In the first row of Table 3, we report calculations associated with the Observed US Duopoly, all calculated at the divided equilibrium point. The final column reports our bound, which is extraordinarily large: $\partial ln(U_i)/\partial ln(v_i) > 25$. Such a large elasticity is implausible,[39] so we conclude that our model is consistent with the multiyear persistence of the US observed divided duopoly. How our model produces this result is easily understood. Our estimates let us calculate (not bound) $J_vJ_{\widehat{r}}(U)$ in elasticity form. The table reports this in elasticity form, $\partial\ln(v_p)/\partial\ln(U_p)$. For both Android and iOS this is small, around 0.02. The behavior of developers in supplying applications to platforms, *measured in terms of the applications' attractiveness to users*, is very damped. Accordingly, the response of users to the availability of applications would need to be explosive for the divided equilibrium to be unstable.

Of course, the Observed US Duopoly bound is high in part because because both platforms' installed bases are so large. To understand the implications of our estimates for stability if there were a third, smaller platform or if the market size were much smaller, we calculate two further bounds.

One of these bounds is for a smaller market size, 1/6 that of the US. We evaluate the Jacobian and

38. Our procedure for estimating N_X is laid out in the Appendix VIII.D.4.

39. For our bound on η_i^i to be violated requires a strength of user reaction such that, for example, if the eBay app were unavailable on iPhones, over a quarter of iPhone users would switch to Android phones. Such a large platform demand elasticity is inconsistent with the widely held view among industry participants that iPhones have been successfully differentiated from (most) Android phones, all but the most expensive ones – see Bresnahan and Greenstein (2014). The available estimate of the *price* elasticity of demand for iPhones, from Sinkinson (2020), is less than 1. However, it is likely not reasonable to assume that the price elasticity is a good quantitative estimate of other demand elasticities. Users may respond less to handset prices than to other elements of the platform surrounding the handset. Handset prices are often not charged directly to the user, but instead are paid as a part of a cell services subscription. Handsets are a durable good, and financed in opaque ways. Still, the best quantitative evidence about demand elasticities does not suggest an explosive user response.

Table 3: Supply Elasticities and Stability Bounds

Evaluated at:	v_d	v_i	v_3[c]	$\frac{\partial \ln(v_d)}{\partial \ln(U_d)}$	$\frac{\partial \ln(v_i)}{\partial \ln(U_i)}$	$\frac{\partial \ln(v_3)}{\partial \ln(U_3)}$	Bound on η_i^i
Observed US Duopoly [a]	14.90	13.51		0.022	0.020		25.39
Small-Market Duopoly[b]	13.03	11.92		0.179	0.168		3.100
Triopoly; smaller platform added to observed duopoly[c]	14.90	13.51	11.26	0.022	0.020	0.236	3.104

[a] The"Observed Duopoly" row is evaluated at historical U_d, U_i. Supply and v_p are based on our estimates. The assumed logit has consumers choosing only between iOS and Android. Shares are 0.559 and 0.441.

[b] The "Small-Market Duopoly" has two platforms with the same shares as the historical duopoly, but with U_d and U_i 1/6 as large. We hold all the economic parameters fixed.

[c] The "Triopoly; smaller platform added to observed duopoly" row leaves U_i and U_d at observed levels and assigns all other observed smartphones (Symbian, Windows Phone, Blackberry) in use in January 2013 to U_3. The resulting shares are 0.530, 0.418, and 0.052. The artificial Platform 3 occurs only in triopoly situations and is endowed with iOS economic parameter values.

calculate the bounds in the Small-Market Duopoly row using the same platform market shares as in the observed historical duopoly. We use all of the same parameter estimates. What changes are the $\widehat{r}$ values which rise to reflect the smaller installed base and, of course, the possibility that v_p is more responsive to U_p in a smaller market. The values for $f_{Xp}(r)$ at those higher thresholds are within the range of our data. We have many observations of applications with r above and below the higher thresholds. If we tried a bound moving the other direction, to a market even larger than the US duopoly, it would take us outside the range of our data, so we do not attempt this.

The differences between the Observed US Duopoly and the Smaller Market Duopoly results illustrate the logic of our model given the applications heterogeneity observed in the US market. The bound on η_i^i is much lower, just over 3, making instability of the divided equilibrium in a smaller market imaginable. The changed bound arises because the estimated quantities $\partial \ln(v_p)/\partial \ln(U_p)$ are significantly larger evaluated at this point, over 8x as large as in the observed duopoly row for each platform. Another advantage of computing equilibrium quanta for a very different market size is that we can see the supply elasticities over a wide arc. Much of the supply of applications, measured in terms of attractiveness to users, is inframarginal. If we reduced the market size by 6/6, v would fall to zero. Reducing market size by 5/6, and thus reducing each platform's installed base by that proportion, reduces v by about 12%. Over a wide range, the distribution of application heterogeneity is star-dominant and the supply behavior of the star applications is inframarginal. Of course, the inframarginal suppliers at large market size must become marginal suppliers at some smaller size – the supply of applications switches from inelastic to elastic at some point as market size falls.

We can learn more about the logic of our economic model, evaluated at our estimates, by looking at the

Triopoly row of Table 3. Here we consider stability when a third, smaller, platform is added to the Observed Duopoly. We leave U_d and U_i at their observed levels, and give this third platform an installed base (U_3) 1/8 the size of the iOS installed base at the Observed Duopoly. We give this third platform the same parameters as iOS. Once again, we evaluate the (in)stability index with the same parameters but at a different point. Here, too, the stability bound is much smaller, about 3.1, so once again, tipping seems plausible. Here the bound is based on the eigenvalues of a 3×3 matrix because there are three platforms. Note that only one of the $\partial \ln(v_p)/\partial \ln(U_p)$ differs from the Observed Duopoly row of the table, the one for $\partial \ln(v_3)/\partial \ln(U_3)$. This is by construction, but it also informs our interpretation of the Triopoly results. The small platform is likely to tip out. Like the Small-Market Duopoly calculation, the Triopoly calculation draws our attention to the possibility that applications heterogeneity makes platform supply behavior, measured in terms of the value of the applications to consumers, much more explosive with a smaller installed base for any platform.

The smaller-market results can be interpreted as applying to a country smaller than the US, poorer than the US, or both. The product of per-customer economic returns to app suppliers, μ, times the size of the market, U_M, needs to be 1/6 the value of the same product in the US for the results to apply. Also, the other elements of supply and demand for apps and platforms would need to be like those in the US. At a minimum, results show that a smaller market with a similar distribution of application attractiveness to users as in the USA, i.e., a star-dominant one, could have an unstable, rather than aw stable, divided equilibrium.

Interpreting the Triopoly results as applying to the experience of Windows Mobile also involves additional unverified assumptions. Here, we continue to examine the US. However, when we assign the iOS parameters to our third, hypothetical, platform, we are clearly assuming a very capable third platform. At the minimum, our analysis suggests that a third US platform might need to grow significantly larger than WinMo ever did, i.e., to create an approximately evenly divided Triopoly, for equilibrium to be stable rather than for the smallest platform to face a powerful tendency to tip out. That would have required a substantially larger investment in attracting mobile developers and users, increased by the need to overcome platform disadvantages, than Microsoft ever undertook.[40]

VII Conclusion

The market structure in smartphone applications development platforms in the US, the largest application development platform market (as yet) ever seen, is surprising. Rather than tipping to a dominant platform, the market has stayed at an approximately evenly divided duopoly. This is not because there are no positive feedback forces benefiting larger platforms and penalizing smaller ones. Smaller platforms have tipped out

40. Recall Microsoft, a highly successful development platform firm, spent multiple $Billions.

of the US, including an effort by a firm expert in platform supply which lost billions on the effort. Market structure outcomes in smaller countries are more consistent with tipping. A single change to traditional platform models can explain all these outcomes. Heterogeneity in applications attractiveness to users with a star-dominant distribution renders the supply of applications to a large platform inelastic. The mechanism is simple: The most attractive applications supply even a moderately-sized platform; their supply behavior is inframarginal if a platform has many users. With one side of the positive feedback loop damped, the entire loop can be explosive only if users respond very elastically to changes in available applications on a platform. The same mechanism not only does not contradict, but implies, more elastic supply of applications to a smaller platform or in a smaller market.

We have estimated the distribution of applications attractiveness to users across both iOS and Android apps, resolving the selection and truncation issues associated with our data supplier's sampling frame and with endogenous supply of applications to platforms. We have also estimated an application supply to platforms model, taking into account the problem that demand is imperfectly forecastable by developers *ex ante* entry. The estimated attractiveness heterogeneity distribution, like the empirical CDF of the raw data (with its selection and truncation problems), is star-dominant. These estimates let us quantify the elasticity of the supply of applications to platforms. It is quite inelastic at the historically large installed bases of the US duopoly. The elasticity of supply would, however, be far smaller in a counterfactual smaller country duopoly or with a smaller third platform in the US. Thus, our estimates can resolve the mixture of stability and tippyness found in smartphone platform markets by quantifying the elasticity of applications supply at different platform installed bases.

Our results explain the evident stability of platform market equilibrium in the US, itself novel and important. In conclusion we note two other implications. The form of smartphone applications heterogeneity, i.e., heterogeneity in attractiveness to users, is characteristic of many consumer markets in entertainment, media, and games, as is the star-dominant shape of the distribution of heterogeneity. The platform market organization is used more and more in these markets, so consideration of models like ours is timely. More generally, the form of heterogeneity of platform market participants and the shape of the distribution of heterogeneity changed stability analysis dramatically; this appears to be an important general point about platform economics, not limited to heterogeneity in applications' attractiveness to users nor to star-dominant distributions. Our analysis has only begun the exploration of this new area.

VIII Appendix

In this appendix we state more carefully and prove the theoretical results of Section III. The traditional version is covered in Section VIII.A, and the applications heterogeneity version in Section VIII.B. Section VIII.C discusses the ways in which our empirical model is less restrictive than the theoretical model. Finally, this appendix lays out the formulae for (in)stability indexes in our empirical model, the assumptions behind our bounds, and provides some background tables in Section VIII.D.

VIII.A The Folk Theorem

We begin with the model defined by (1), (2), (3), and (4). The main economic assumptions are noted in the text. Here we show that under a few regularity conditions, the folk theorem result follows. Further, under symmetry across platforms, the possible values of the equilibrium correspondence are (a) one stable equilibrium, or (b) 3 equlibria ordered stable/unstable/stable.

Regularity conditions: Assume that both F_u and F_a are continuous in all their derivatives, are symmetric around zero, and that F_u has infinite support. Each is strongly unimodal in the sense that $f_a(x) < f_a(y)$ and $f_u(x) < f_u(y)$ whenever $|x| > |y|$.

Recalling $\Delta_\pi \equiv \mu(U_1 - U_2) - C_1 + C_2$, rewrite (2):

$$\text{(A20)} \quad N(U_1, U_M - U_1) = [F_a(\mu(U_1 - (U_M - U_1)) - C_1 + C_2) \;\; 1 - F_a(\mu(U_1 - (U_M - U_1)) - C_1 + C_2)] \,.$$

Recalling $\Delta_V(N) \equiv \gamma(N_1 - N_2) + \gamma_1 - \gamma_2$ rewrite (4) substituting in (A20):

$$\text{(A21)} \qquad \chi_1(U_1) = U_M \times F_u(\gamma(2F_a(\mu(U_1 - (U_M - U_1)) - C_1 + C_2) - 1) + \gamma_1 - \gamma_2).$$

We have $\chi_1'(U) > 0$ and $N_1(U_1, U_M - U_1)$ increasing in U_1. It is a positive feedback model.

Infinite support implies $0 < F_u(y) < 1$ for finite y. Thus it is immediate from (A21) that $\chi_1(0) > 0$ and $\chi_1(U_M) < U_M$. Also, $\chi_1(U_1)$ is continuous and has continuous derivatives. These geometric properties lead directly to a number of results.

- There is a stable equilibrium. Continuous χ_1 must pass from above the 45° line to below it.
- Whether equilibrium is unique or not, the equilibria with the largest and smallest U_1 are always stable.
  - Proof: Let $U^e > 0$ be the smallest equilibrium. $\chi_1(0) > 0$ and $\chi_1()$ continuous, so $\chi_1(y) > y \;\forall\; y < U^e$. Thus $\chi_1()$ crosses the 45° line from above at U^e. A symmetric argument applies to the largest equilibrium. See Figure 1.
- Generically in the parameters C_p and γ_p, the number of equilibria is odd.
- If there is a strictly unstable equilibrium, there are at least three equilibria. This follows because the

largest and smallest equilibria are stable.

1. Outside the symmetric case, there can be two equilibria, where one is strictly unstable and is a point of tangency between χ_1 and the 45 degree line. The equilibrium correspondence is not continuous at such points.

VIII.A.1 Further Results Under Symmetry

The central role of instability of divided equilibrium under symmetry leads us to examine more results in this case. For this subsection, assume symmetry, i.e., $C_1 = C_2$ and $\gamma_1 = \gamma 2$, unless otherwise noted. The slope of $\chi_1(U_1)$ can now be written

(A22) $$\chi_1'(U_1) = 2 * \gamma * f_u(\Delta_V(N)) * U_M * \mu * f_a(\Delta_\pi(U)).$$

If $U_1 = U_2 = U_M/2$, (A22) yields

(A23) $$\chi_1'(U_M/2) = 2 * \gamma * f_u(0) * U_M * \mu * f_a(0).$$

– There is a divided equilibrium. Proof: $\chi_1(U_M/2) = U_M \times F_u(\gamma(2F_a(\mu(0)) - 1) + 0) = U_M * .5$.

– The divided equilibrium is stable iff:

$$\text{Stability:} \qquad 2 * \gamma * f_u(0) * U_M * \mu * f_a(0) < 1.$$

– The divided equilibrium is *more tippy / less stable* if γ, μ, or U_M is larger. The γ and μ results simply mean stronger network effects lead to more tippiness. For any development platform, marginal profits for developers will include $\mu \times U_M$, so the slightly less familiar result that larger markets are more tippy follows immediately.

– The divided equilibrium is *more tippy / less stable* if $f_u(0)$ or $f_a(0)$ increases, i.e., if the distribution of tastes or profits is closer to degenerate.

– $\chi_1'(U)$ is maximized at the divided equilibrium $U = U_M/2$, is strictly decreasing for $U > U_M/2$ and strictly increasing for $U < U_M/2$.

– The only possible equilibrium correspondences are (a) a unique, stable divided equilibrium and (b) three equilibria in the order stable/unstable/stable.

  * If $2 * \gamma * f_u(0) * U_M * \mu * f_a(0) < 1$, then $\chi_1'(U) < 1 \; \forall \; U$, and the stable divided equilibrium is unique.
  * If $2 * \gamma * f_u(0) * U_M * \mu * f_a(0) = 1$, then $\chi_1'(U) = 1$ only at $U = U_M/2$ and equilbrium is unique.

  * If $2 * \gamma * f_u(0) * U_M * \mu * f_a(0) > 1$, then there is a divided strictly unstable equilibrium. There must be at least one stable equilibrium above and another below $U_M/2$. Since $\chi_1'(U)$ is monotonically declining (increasing) for $U > U_M/2$ (for $U < U_M/2$), there is only one instance of $\chi_1'(U) = 1$ for $U > U_M/2$ (for $U < U_M/2$).
  * Multiple equilibria alternate between stable and unstable as U_1 increases.
- The equilibrium correspondence is everywhere continuous (i.e., both upper- and lower- hemi-continuous).

- Since the equilibrium correspondence is continuous around the symmetric case, for economic primitives close to symmetry and close to a symmetric case with three equilibria, there are also three equilibria with the divided equilibrium not at the symmetric point.

VIII.B Heterogeneous Application Variant

We examine the model defined by (7), (8), (9), and (10) with $\Delta_V = \gamma(v_1 - v_2) + \gamma_1 - \gamma_2$. Assume that F_u is continuous in all its derivatives and has infinite support, is symmetric around zero, and is strongly unimodal in the sense defined above. Assume that F_r has support on $[0, 1]$. The bounded support for r_a implies that no application finds it profitable to supply platform p for $U_P < U_p^L \equiv C_p/\mu$. To avoid endlessly (and uselessly) checking boundary cases, we assume $U_M > U_1^L + U_2^L$. Application supply to platform p is

(A24)
$$V_p(U_p) = \begin{cases} 0 & \text{for } U_p < U_p^L, \\ \int_{\widehat{r}_p}^1 t f_r(t) dt & \text{for } U_p \geq U_p^L \text{ where } \widehat{r}_p = C_p / (\mu U_p) . \end{cases}$$

User demand for platforms, reprinting (8), is

(A25)
$$U(v) = U_M \times [F_u(\Delta_V(v)) \;\; 1 - F_u(\Delta_V(v))] .$$

(A25), (A24), and $U_2 = U_M - U_1$ let us once again define the function $\chi_1(U_1)$.

$$\chi_1(U_1) = U_M F_u(\gamma(V_1(U_1) - V_2(U_M - U_1)) + \gamma_1 - \gamma_2).$$

Thus χ_1 is continuous, upward-sloping, has $\chi_1(0) > 0$ and, symmetrically, $\chi_1(U_M) < U_M$. χ_1 is smooth except at $U_1 = U_1^L$ and $U_1 = U_M - U_1^L$, where its derivative is not continuous. These geometric properties mean that this model has some of the same implications as the folk theorem version: There is always a stable equilibrium. Whether equilibrium is unique or not, the equilibria with the largest and smallest U_1 are always stable. The neighbors of an unstable equilibrium, if one exists, are stable.

Under symmetry, i.e., $C_1 = C_2$ and $\gamma_1 = \gamma 2$, there is a divided equilibrium with $U_1 = U_2$. Either that equilibrium is unique, or the equilibrium correspondence is as in Figure 3, with five equilibria.

Under star-dominance of $f_r()$, $V_p'(U_p)$ is decreasing and convex over the range $U_p > U_p^L$. It is not globally convex: while $V_p(U_p)$ is continuous at U_p^L, $V_p'(U_p)$ is discontinuous at that point. On the left, $V_p'(U_p^L - \epsilon) = 0$, while on the right, $V_p'(U_p^L + \epsilon)$ approaches its global maximum under star-dominance.

Convexity of $V_p'(U_p)$ over the range where there is application supply to platform p is easiest to see by writing it as a function of $\widehat{r}_p$ only. Using (A24), $V_p'()$ for $U_p > U_p^L$ is

$$V_p'(U_p) = \partial v_p(\widehat{r}_p(U_p))/\partial U_p = -f_r(\widehat{r}_p)\widehat{r}_p \;\times\; -C_p/\left(\mu U_p^2\right) = f_r(\widehat{r}_p)\widehat{r}_p * \widehat{r}_p^2 * \mu/C_p.$$

We see that $V_p'(U_p)$ is positive and, if $f_r(r) * r$ is increasing in r (implied by star-dominance), $V_p'(U_p)$ is increasing in $\widehat{r}_p$ and thus decreasing in U_p. Thus, under star-dominance, $V_p'(U_p)$ is convex.

An increasing, convex $f_r(r)r$ is sufficient for convexity of $V_p'(U_p)$ but not necessary. Abbreviating $h(r) = f_r(r)r$, a sufficient condition is $r12h(r)+8r^2h'(r)+r^3h''(r) > 0$. Since this condition has no obvious economic interpretation, we do not pursue it. Our empirical inquiry focuses on verifying that the necessary condition holds.

Turning now to the stability analysis, we note that an arbitrary point:

$$\text{(A26)} \qquad \chi_1'(U_1) = f_u(\Delta_V) * U_M * \gamma\left[V_1'(U_1) + V_2'(U_M - U_1)\right].$$

Outside the interior range, $U_1 > U_M - U_1^L$ or $U_1 < U_1^L$, and one of the terms in square brackets is zero.

To consider stability of a divided equilibrium, assume symmetry, i.e., $C_1 = C_2$ and $\gamma_1 = \gamma_2$. There is a symmetric equilibrium with $U_1 = U_2$, $\widehat{r}_1 = \widehat{r}_2 = \mu/(C \times U_2)$, $v_1 = v_2$, and $\Delta_V = 0$. Symmetry also implies that $V_1(U) = V_2(U) = V(U)$, so we can write

$$\chi_1'(U_1) = f_u(0)U_M\gamma[V'(U_1) + V'(U_M - U_1)].$$

Since $V'(U)$ is a convex function, the term in square brackets attains a local *minimum* at $U_1 = U_M - U_1$.

This result means that there can be a stable divided equilibrium without ruling out the possibility of multiple equilibria. Of course, if $f_u(0)$ is sufficiently large, the divided equilibrium is unstable.

The economics behind Figure 3 can now be easily understood. Suppose that the inequality defining convexity for V', $V'(U_1) + V'(U_M - U_1) - 2V'((U_1 + U_M - U_1)/2) > 0$, is quantitatively large for some U_1. Then the supply of applications to platforms is far less responsive to installed base at the divided equilibrium than at U_1. So instability can arise, strongly, away from the divided equilibrium for the same reason that the divided equilibrium is stable. The largest and smallest equilibria must be stable. In our model, the finite support of the distribution of applications heterogeneity leads to U_p^L and this outcome.

We next look at the comparative statics of the divided equilibrium. Symmetry lets us simplify the expression for the (in)stability index at the divided equilibrium (denoting $\widehat{r}(U_M/2)$ by $\widehat{r}$):

$$\chi_1'(U_1) = f_u(0) * \gamma * U_M * \Big[2V'(U_M/2)\Big]. \quad \text{(A27)}$$

Noting that $V'(U_M/2) = -f_r(\widehat{r})\widehat{r} \;\times\; -C/\left(\mu(U_M/2)^2\right)$, we have

$$\chi_1'(U_1) = f_u(0) * \gamma * 2 * \Big[f_r(\widehat{r})\widehat{r} \;\times\; C/(\mu(U_M/4))\Big] = f_u(0) * \gamma * 2\Big[2\widehat{r}^2 f_r(\widehat{r})\Big], \quad \text{(A28)}$$

where the last equality uses $1/(U_M/2) = (\mu/C)\widehat{r}$.

We use this expression to get results about the role of user heterogeneity and of market size.

- As we change F_u, the divided equilibrium has the same $\widehat{r}$. Thus it is unstable for all $f_u(0) > 1/(4\gamma\widehat{r}^2 f_r(\widehat{r}))$ but otherwise stable.
- Under symmetry with star-dominance, at the divided equilibrium the (in)stability index is decreasing in market size. This is easy to see as (A28) is clearly increasing in $\widehat{r}$, i.e., decreasing in $U_M/2$.
- Indeed, under symmetry with star-dominance, there is a U_M^* such that for all $U_M > U_M^*$, the symmetric equilibrium is stable. This is easy to see as (A28) is the product of $\widehat{r}$ and of $\widehat{r} f_r(\widehat{r})$, which must approach a bounded infimum as $\widehat{r}$ nears zero (as $U_M/2$ grows). The supply elasticity of apps to platforms becomes arbitrarily small for large enough U_p.

The comparative statics in market size are the opposite of those implied by the folk theorem.

VIII.C Empirical Model Less restrictive than theory – two quantitatively important and two unimportant differences

The theoretical model gives developers a perfect pre-entry forecast of r_a. The empirical model dampens the supply response by giving developers an imperfect signal. This is quantitatively significant. λ is typically about 0.64, meaning that while the signal is more often than not good, about a third of the time the signal is uninformative.

The theoretical model has a single $f_r()$. The empirical model conditions $f_r()$ on observables about the developer and the application. Conditional on the distribution of X, this is a distinction without a difference within platform. In our estimates, it is also a small difference across platforms, though it need not have been. See discussion of Figure 9.

The theoretical model endows each application with the same r_a on all platforms. In the empirical model estimates, r_{ai} and r_{ad} are not identical. However, because the parameters that condition $f_r()$ on observables are very similar between platforms, even the independent component of r_a has very similar distribution on the two platforms.

The real-world equilibrium is not symmetric. U_d is about 5/4 the size of U_i. In our estimates, this difference is somewhat offset by a higher μ/C on the iOS platform. Thus the supply of apps is similar but not identical across platforms. At the historical equilibrium point, $v_i < v_d$ in our estimates (and in the raw data.) The historical divided equilibrium is thus at a point where the slope of the supply of applications to platforms is not exactly at a local minimum in $U_d, U_M - U_d$, but it is near one and at a low value.

VIII.D Empirical Jacobian Details

VIII.D.1 $J_{\widehat{r}}$

In the empirical model, X takes on multiple values. This means that the supply threshold function $\widehat{r}(U)$ is from $\mathbb{R}^P$ to $\mathbb{R}^{P*dim(X)}$ where P is the number of platforms and $dim(X)$ is the number of values of X. Thus, we are going to define a value of $\frac{\partial \widehat{r}_{Xp}}{\partial U_p}$ for each X and p. $\frac{\partial \widehat{r}_{Xp'}}{\partial U_p} = 0$ for $p' \neq p$.

(A29)
$$\widehat{r}_{pa} = \frac{\kappa_{Xp}}{\lambda_{Xp}\, U_p} - (1-\lambda_{Xp})/\lambda_{Xp} \times \mathbb{E}[\,\overline{r}_{pa} \,|\, X, \theta\,],$$

which implies

(A30)
$$\frac{\partial \widehat{r}_{Xp}}{\partial U_p} = -\frac{\kappa_{Xp}}{\lambda_{Xp} U_p^2}.$$

VIII.D.2 J_v

First, we need to calculate the portion of v_p that comes from a given X and take into account the gap between developer supply to a platform (based on their signal) and how much they contributed to user app demand (based on realized r). This means that the contribution from developers of type X to platform p is

(A31)
$$v_{Xp}(\widehat{r}_{Xp}) = N_X \left[\lambda_{Xp} \int_{s=\widehat{r}_{Xp}}^{1} s f_{Xp}(s) ds \ + \ (1-\lambda_{Xp}) Pr(s > \widehat{r}_{Xp}) E[\overline{r}_{Xp}] \right],$$

where N_X is the number of developers who might write an attractive app of type X, $f_{Xp}(s)$ is as defined above, and the two terms in the large brackets represent the contributions of applications that do and do not realize their signaled reach, respectively. We sum these over all the values of X to get $v_p(\widehat{r}_p)$.

Accordingly, the element of J_v for platform p and app type X is

(A32)
$$N_X \left[-\lambda_{Xp} * \widehat{r}_{Xp} * f_{Xp}(\widehat{r}_{Xp}) \ + \ (1-\lambda_{Xp}) E[\overline{r}_{Xp}] f_{Xp}(\widehat{r}_{Xp}) \right].$$

VIII.D.3 J_D

For empirical stability bounds, we assume a one-parameter logit model and put a bound on that parameter. We write:

$$\bar{v}_p + \gamma v_p,$$

where $\bar{v}_p$ captures all the factors that lead users to pick platforms other than app availability, such as the price and quality of the platform's devices. We will always be evaluating the elasticities at fixed shares, so the $\bar{v}_p$ disappear into the shares, leaving only γ. Using σ for shares, we have

$$J_D = \gamma U_M \begin{pmatrix} \sigma_1(1-\sigma_1) & \sigma_1\sigma_2 & \dots \\ \sigma_2\sigma_1 & \ddots & \\ \vdots & & \ddots \end{pmatrix}.$$

We report η_i^i, the elasticity of demand for iOS with respect to v_i, where $\eta_i^i = \gamma v_i/(1-\sigma_i)$.

The mechanics of the bounding calculation itself is simple. We calculate J_D under the assumption that $\gamma = 1$, then find the largest eigenvalue of $J_D J_v J_{\widehat{r}}$. 1/that eigenvalue is the bound on γ.

VIII.D.4 Population of Capable Developers

We estimate the number of app developers of each observable type X, N_X, as

$$N_X = n_X / Pr(S_i > 0 \;\vee\; S_D > 0 \,|\, X),$$

where the numerator n_X is the number of apps of type X observed in our sample and the denominator is the probability that an app is observed. The sum of n_X is 1,044 (our sample size) and the sum of N_X is just under 2,646. We think of this as an estimate of the population of developers who had an app potentially suitable for mass market distribution and sufficient resources to market it, not of all developers.

The striking thing about Table 4 is that the probability of selection for a Mobile Era (O) firm's app is much lower at 26% than for an app from firms of the other two, "established" eras.

Table 4: Sample Selection By X

X^a	n	N	Pr()
LHG	48	89.8	0.5343
OHG	177	824.1	.2148
FHG	22	41.6	0.5287
LTG	24	37.0	0.6493
OTG	3	7.6	0.3945
FTG	53	82.1	0.6454
LHY	151	254.0	0.5944
OHY	257	841.7	0.3053
FHY	76	128.8	0.5898
LTY	80	115.8	0.6910
OTY	1	2.2	0.4609
FTY	152	221.0	0.6878
All L[b]	303	496.6	0.610
All O	438	1675.6	0.261
All F	303	473.6	0.640

[a] X is the value of X in the notation of Figure 9. n is the number of sample points for that value of X. $Pr()$ is the probability that an application of type X will be observed $\Pr(S^*_{ia} + S^*_{da} > 0 \mid X, \theta)$ N is the number of unselected applications of type X predicted by $n/Pr()$. For example, the first row points to X=LHG, Game apps from Online-Era firms that are Privately Held; there are 48 of these in our sample, about 90 of these in the population of capable developers, and a 53% chance of an application of this type appearing in our sample.

[b] The rows All L, All O, and All F aggregate the rows above them. For example, the All L row shows that we have 303 apps from Online Era firms, and that an Online-Era firm has a 61% chance of being observed. This clarifies the definition of "potential entrant" in our model. There are many millions of developers who have written an app. Economically, a potential entrant is one with the resources 1) to make an app which appeals to a meaningful number of users and 2) to market the app and bring it to those users' attention.

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