FUTURES MARKET BACKWARDATION UNDER RISK NEUTRALITY

TIMOTHY F. BRESNAHAN and PABLO T. SPILLER*

J.M. Keynes first introduced the theory of "normal backwardation" in futures markets. In the language of (British) commodities markets, a "backwardation" is an excess of the spot price over futures prices. As is well-known, Keynes suggested that this might be explained as a risk premium. Less well known is that Keynes actually proposed two distinct theories of backwardation.

Of these two theories of backwardation, the latter has recently received much attention. The purpose of this paper is to formalize Keynes' first theory, his "liquid stocks" theory, with an eye to its eventual empirical test.

We follow the recent formalizations of the risk premium theory by assuming the existence of perfectly competitive asset markets. To emphasize the differences between the two theories, however, we assume that there are well-funded risk neutral investors. Thus, risk premia cannot explain backwardation under our assumptions. Instead, backwardations arise because of interactions between equilibrium in the commodities exchange, both in spot and futures trading, and the production, consumption and storage decisions taken on the real side of the economy.

I. INTRODUCTION

J.M. Keynes first introduced the theory of "normal backwardation" in futures markets.[1] In the language of (British) commodities markets, a "backwardation" is an excess of the spot price over futures prices.[2] As is well-known, Keynes suggested that this might be explained as a risk premium. Less well known is that Keynes actually proposed two distinct theories of backwardation:

> If there are no redundant liquid stocks, the spot price may exceed the forward price (i.e., in the language of the market there is a 'backwardation'). If there is a shortage of supply capable of being remedied in six months but not at once, then the spot price can rise above the forward

*Department of Economics, Stanford University, and Hoover Institution, Stanford University, respectively. This research was partially supported by the Center for the Study of Futures Markets, Columbia Graduate School of Business.

1. John Maynard Keynes, [1930, Chapter 29].

2. Two possible confusions about the word "backwardation" can arise. First, we adopt the convention of British floor traders of using "backwardation" to mean no more than the occurrence of spot price higher than futures price. In the United States, a backwardation is called an "inverted market." Adding to the confusion is the term "normal backwardation," which is typically used to denote a specific explanation of backwardations—the risk premium theory described below.

> price to an extent which is only limited by the unwillingness of the buyer to pay the higher spot price rather than postpone the date of his purchase . . .
>
> But it is not necessary that there should be an abnormal shortage supply in order that a backwardation should be established . . . the spot price must exceed the forward price by the amount which the producer is ready to sacrifice in order to 'hedge' himself . . . [3]

Of these two theories of backwardation, the latter has recently received much attention.[4] The formalization of this "risk-premium" theory has led to some interesting empirical work.[5] The purpose of this paper is to formalize Keynes's first theory, his "liquid stocks" theory, with an eye to its eventual empirical test.

We follow the recent formalizations of the risk premium theory by assuming the existence of perfectly competitive asset markets.[6] To emphasize the differences between the two theories, however, we assume that there are well-funded risk neutral investors. Thus, risk premia cannot explain backwardation under our assumptions. Instead, backwardations arise because of interactions between equilibrium in the commodities exchange, both in spot and futures trading, and the production, consumption and storage decisions taken on the real side of the economy. The link is exactly that suggested by Keynes: stocks of a commodity can either be 'liquid'—i.e., available to be held as an asset—or they can be tied to a production/consumption process.

Our model is also closely related to theories of the "supply of storage" extensively studied in connection with agricultural commodities.[7] In particular, our model has supply and demand uncertainty, but fluctuations in the value of the commodity can be smoothed out by carrying inventories forward.[8] We assume, in the useful language of Blau [1944] that the "utility created by carrying stocks of commodities . . . is a derived utility, i.e., it depends on the utility of the finished product to consumers (e.g., in the case of carrying wheat from September to March, the utility of March wheat to consumers)." That is to say, we assume there is no "convenience yield," no direct benefit to consumers of having an inventory on hand. Thus, the

3. Page 143. Page references are to the 1959 printing.

4. See Breeden [1980], Cox, Ingersoll and Ross [1981], Jagannathan [1983], Grauer and Litzenberger [1979], Richard and Sundaresan [1981], and French [1983].

5. See especially Richard and Sundaresan [1981] for the result that the sign of the real risk premium in a futures price depends on whether the future is a consumption hedge. The empirical evidence on the signs of real risk premia is mixed. Futures in some commodities, at some times, have had real risk premia consistent with backwardation, while others have not. See Breeden [1980], Grauer and Rentzler [1980], and Gray [1961]. The empirical evidence on real risk premia whose signs change over time, so that both backwardation and contango at different times can be explained, is even thinner. See Jagannathan [1983].

6. Alternative explanations, based on the microdynamics of imperfect (thin) asset markets, exist as well. See Gray [1961].

7. See, especially: Telser [1958] and Peck [1977], Vol. II.

8. Clearly there are other motives such as production taking time, as in Reagan [1982]. See also Carlton [1977] and Section 2 of Peck [1977].

storage we seek to model is the holding of stocks of commodity purely as an asset—the holding of liquid stocks. We treat inventories yielding a convenience yield as being consumed.[9] The purpose of this strong dichotomy is not to deny the empirical importance of convenience yields, but rather to focus attention on liquid stocks.

Our results follow from an obvious observation about the supply of storage. Whatever returns are required to induce different *positive* amounts of holding of liquid stocks, there is no return which leads the asset market to hold *negative* inventories. We call the event liquid stocks of zero a "stockout." We show that in an economy with uncertainty, there is always a positive probability of a stockout at some time. Further, stockouts induce backwardations in exactly the sense and for exactly the reasons Keynes suggested.

The liquid stocks theory and the risk premium theory have different empirical implications, as we shall show. Both are advanced as explanations of backwardations—excesses of the spot price *now* over the futures price now. The risk premium theory explains backwardations by "normal backwardation"—the upward bias of the futures price as a predictor of the spot price *in the future*. Tests of the risk premium theory have focused on attempts to measure this bias. In the liquid stocks theory, futures are unbiased predictors of future spot prices because of risk neutrality. However, stockouts lead to a situation in which current *spot* prices (adjusted for carrying charges) are upward biased predictions of future spot prices. These differences will show most clearly when demand is subject both to permanent and temporary shocks. A permanent shock will raise price both now and in the future, while a transitory shock raises price only now. If a transitory shock is sufficiently large, it will cause both a stockout and a backwardation. The last section of this paper presents some preliminary evidence in favor of the empirical importance of the liquid stocks theory.

II. A SIMPLE MODEL OF CONSUMPTION, PRODUCTION, AND STORAGE

This section specifies the behavior of producers, consumers and storers of a single commodity. The supply of storage facilities (such as warehouses) has constant marginal costs. The only assets available are bonds with exogenous interest rates, the single stored commodity, and futures in that commodity. No risk aversion exists. However, future demand and supply are uncertain, and negative storage is not allowed.

Because of the possibility of storage, consumption and production need not match at every instant; discrepancies can be met by drawing down or increasing the inventory of stored commodity. The behavior of both pro-

9. See Blau for an argument that convenience yields are unimportant for the kinds of commodities traded on exchanges. Empirical evidence on the extent of genuinely liquid stocks is quite difficult to obtain. "Work in Process" inventories are clearly nonliquid, but the accounting dividing line defining such inventories is clearly difficult to set precisely.

ducers and consumers of the commodity is summarized in Q_t, the excess of quantity demanded over quantity supplied at time t. This is determined by

(1) $$Q_t = g(\tilde{A}_t, P_t) \quad \text{(Instantaneous Excess Demand)},$$

where P_t is the price at time t, and $\tilde{A}_t$ is a random term, the only source of uncertainty in the model. We assume strictly downward-sloping demand, with slope bounded away from zero:

$$\partial g/\partial P_t < \gamma < 0.$$

Let $f_t(\tilde{Q}_\tau \parallel P_t)$ for $t < \tau$ be the conditional density function of excess demand based on all information at time t. We assume

(A1) $f_t(Q_\tau \parallel P_t)$ is an atomless density function and $f_t(Q_t \parallel P)$ has

$$\lim_{P\to\infty} E[\tilde{Q}_t \parallel P] = -\infty .$$

This assumption rules out lumps of probability in the distribution of demand and permits arbitrary serial correlations in demand. The notations $E_t[\cdot]$ and $Pr_t(\cdot)$ denote expectation and probability conditional on all time-t information, respectively.

The marginal physical cost of storing one unit of the commodity from time t to time $t+1$ is c_t, which does not include the interest carrying charges, to be discussed below. Inventories of liquid stored commodity, I_t, are measured at the end of period t. Thus we have

(2) $$I_t = I_{t-1} - Q_t \quad \text{(Inventory updating)}.$$

Equation (2) links the asset-holding part of the economy of production and consumption. If the demand function for inventory as an asset is substituted into (2), the result is the excess-supply relation of the asset-holding part of the economy. Equating (1) and (2) then gives the overall equilibrium. Finally, commodities cannot be stored in negative amounts.

(3) $$I_t \geq 0 \quad \text{(Nonnegative Storage)}.$$

The asset-market equilibrium condition has two segments. In one of these, inventories are actually being stored. Denote one plus the interest rate from time t to time $t+i$ by $R_{t,i}$; $R_{t,i} > 1$. Since the stored commodity is an asset, it must earn the competitive rate of return. That is,

(4) $$\text{if } I_t > 0,\ R_{t,1}P_t + c_t = E_t[P_{t+1}] \quad \text{(Arbitrage-Storage)}.$$

The left-hand side of (4) is the total opportunity cost of buying and storing one unit of the commodity, and the right-hand side is the expected revenue from resale. Arbitrage forces them to be equal.

On the other segment of the asset-market equilibrium, no inventory is being stored, and prices must be such that investors would not make an extraordinary return by buying and storing the commodity. That is,

(5) $$\text{if } I_t = 0,\ R_{t,1}P_t + c_t \geq E_t[P_{t+1}] \quad \text{(Arbitrage-No Storage)}.$$

Equations (4) and (5) describe the excess-supply relation of the asset-holding part of the economy.[10]

An arbitrage condition also determines futures prices. A futures contract written at time t for maturity at time τ obliges the buyer and seller to trade one unit of physical commodity at time τ. The price is fixed in the contract at $F_{t\tau}$, but no money changes hands until time τ.[11] Thus arbitrage ensures that

$$F_{t\tau} = E_t[P_\tau] \quad \text{(Arbitrage-Futures).} \tag{6}$$

The rate of interest does not distort the relationship between the expected spot price and the current futures price since no physical storage of commodities takes place in a futures transaction.

Equilibrium

Because of the random $\tilde{A}_t$, equilibrium must specify prices as a function of the realization of $\tilde{A}_t$. In recursive form the equilibrium price function is

$$P_t = P_t^*(\tilde{A}_t, I_{t-1}), \tag{7}$$

I_{t-1} is the state variable, summarizing all information as of $t-1$.

The functions $P_t^*(\cdot)$ must satisfy four sets of equilibrium conditions: short run market clearing (1) and (2), the inventory non-negativity constraint (3), period-to-period asset market equilibrium (4), (5), (6), and a "no bubbles" condition we introduce here. The no-bubbles condition is:

if for any t

$$\lim_{\tau\to\infty} E_t[P_\tau] = \infty \text{ and } \lim_{\tau\to\infty} E_t[I_\tau] = \infty$$

then $\lim_{\tau\to\infty} E_t[Q_\tau] > -\infty$.

That is, the stored stock, price, and instantaneous excess supply cannot all grow without bound. A "bubble" is a situation in which the price is expected to rise forever together with ever-increasing expected contributions ($-Q_\tau$) to the stored stock. We assume such bubbles do not occur.[12]

10. Analogs to (4) and (5) are discussed in Samuelson [1957], Grauer and Litzenberger [1979], and Bresnahan and Suslow [1983]. See also Telser [1958] and Scheinkman and Schechtman [1983] for related models. None of these papers treats the relation between the real economy and futures prices, nor the explanation of backwardations as a result of stockouts. In Telser [1958] the "convenience yield" of stocks lead to positive inventories even when the spot price is expected to fall. Our analysis would be complicated but not fundamentally altered by adding convenience yields.

11. Actually this describes what is called a forward contract in U.S. commodities markets. Futures contracts differ by being "marked to market," which means that capital gains and losses on the future are realized each trading day. Since we are assuming the rate of interest as known and investors as risk-neutral futures and forward contracts are equivalent in our model. (See French [1983]). The situation is further complicated by the fact that futures are not "marked to market" on European commodity exchanges, so that European futures contracts resemble U.S. forward contracts rather than U.S. futures ones.

12. In our partial-equilibrium context it would not be possible to rule out bubbles except by

Footnote 12 continued on next page

III. IMPLICATIONS OF EQUILIBRIUM

This section shows how incomplete production smoothing by competitive storage affects futures prices. We classify short run equilibrium into two regimes, depending on the existence of a stockout, and show that both regimes must occur with positive probability. These results provide a basis for the futures price results. We will use equilibrium methods in the proofs, but the intuition of the results is rooted in the optimality of competitive equilibrium.

Characterization of Short-Run Equilibrium

We establish conditions on the relationship between P_t^*, $E_t[P_{t+1}]$ and storage costs. Define

(8) $$\rho_t = E_t[P_{t+1}^*(\tilde{A}_{t+1}, 0)]$$

the next period expected price when no stocks are stored. By the period-to-period asset-market equilibrium conditions (4) and (5), either stocks are held and expected commodity price rises by the carrying charges, or no stocks are held and expected price rises more slowly. The asset-market equilibrium condition prevents expected price rises larger than the carrying charges.

Stockouts, or occurrences of zero inventory, can occur in two ways. First, the expected price could rise exactly by the carrying charges but current excess demand could just clear the market:

(E1) $$P_t^*(\tilde{A}_t, I_{t-1})R_{t,1} + c_t = \rho_t .$$

Second, the inequality in the top of (5) could be strict. Then investors would be strictly unwilling to hold the commodity stock.

(E2) $$P_t^*(\tilde{A}_{t1}, I_{t-1})R_{t1} + c_t > \rho_t .$$

In either case, we must have

$$Q_t = g(\tilde{A}_t, P_t^*(\tilde{A}, I_{t-1})) = I_{t-1} .$$

Under our assumption that $\tilde{A}_t$ has a continuous distribution, (E1) is a zero-probability event. Thus:

(9) $$Pr_{t-i}(I_t = 0) > 0 \quad <=>$$
$$Pr_{t-i}(P_t^*(A_t, I_{t-1})R_{t,1} + c_t > E_t[P_{t+1}^*]) > 0 .$$

That is, if the probability of a stockout is positive ex ante, then so is the probability of prices rising more slowly than the carrying charges.

These results are most intuitive in light of the optimality of competitive equilibrium. Socially, $I_t \geq 0$ is a constraint on smoothing production or con-

Footnote 12 continued

assumption. See Tirole [1985] on the existence of bubbles. He also shows that unless the economy is stationary or if it grows at a rate less than the long-run real rate of interest, bubbles can appear in general equilibrium models.

sumption over time. If static considerations make the commodity more valuable in the future, storage can move the commodity from less to more valuable uses. But negative storage is impossible, so moving the commodity forward in time to high-value uses is not feasible. The asymmetry in the expected price dynamics is dual to this asymmetry in storage.

The Probability of a Stockout

Even if $I_t > 0$, $Pr_t(I_\tau = 0) > 0$ for some $\tau > t$. Optimality again makes this intuitive: stored commodity is pure social waste if it has no chance of being used later. The proof uses the no-bubbles assumption. Though period-to-period arbitrage was sufficient for the duality of the last section, full intertemporal optimality requires that infinitely large stocks not be carried indefinitely far into the future.

To prove $Pr_t(I_\tau = 0) > 0$ for some $\tau > t$, assume the contrary. Since $I_\tau > 0$ implies expected prices rise by the carrying charges, we have

$$E_t[P_\tau] = P_t R_{t,\tau-t} + \sum_{k=t}^{\tau-1} R_{k+1,t-k-1} \cdot c_k \tag{10}$$

so that expected future spot prices grow without bound. By (A2) this implies

$$\lim_{\tau\to\infty} E_t[Q_\tau] = -\infty . \tag{11}$$

Since prices are growing without bound, expected excess demand grows infinitely large and negative. Therefore

$$\lim_{\tau\to\infty} E_t[I_\tau] = \infty . \tag{12}$$

The stored stock grows without bound as well, which completes the contradiction of the no-bubbles condition.

Results for Futures Prices

The last sections imply the following result about the equilibrium term structure of futures prices:

$$F_{t,\tau+1} \leq F_{t,\tau} R_{\tau,1} + c_\tau \quad \text{for all } t \text{ and } \tau, \tag{13}$$

with

$$F_{t,\tau+1} < F_{t,\tau} R_{\tau,1} + c_\tau \quad \text{for some } t \text{ and } \tau, \tag{14}$$

and

$$F_{t,\tau+1} < F_{t,\tau} R_{\tau,1} + c_\tau \quad \text{iff } Pr_t(I_\tau = 0) > 0. \tag{15}$$

The term structure of futures prices rises no faster than the carrying charges, and must rise slower for some contract length. The only contango allowed under risk neutrality is the storage-cost contango. Relative backwardation (to the storage costs) is expected, and even steeper backwardation is feasible. The term structure of futures prices will show relative backwardation at all future times when there is a positive probability of a stockout.

Moreover, if the term structure of futures prices shows a relative backwardation, then the current spot price plus carrying charges is an upward-biased predictor of future spot prices. Only without backwardation is it an unbiased predictor.

Finally, assume that the $\tilde{A}_t$ are i.i.d. Then the underlying economy is without serial correlation of any form, since the excess demand function $g(\cdot)$ is time-independent. Now observe that (9) implies that a higher inventory at time t means that the probability of stockout at time $t+1$ is lower. Thus, if there is a stockout at time t, the probability of a stockout at time $t+1$ is at a maximum. As a result, backwardations will tend to forecast backwardations, even under the extreme assumption of no underlying serial correlation.

IV. AN EXAMPLE WITH PERMANENT AND TEMPORARY SHOCKS

This section presents an example in which relative backwardation is a normal market phenomenon. This market has three time periods, 0, 1, and 2, so that there is a period (here period 1) for which both the beginning-of-period and the end-of-period inventory is endogenous. Two kinds of shocks occur: a permanent one, which is revealed at time 0, and period-specific temporary shocks. That is

$$g(P_t, A_t) = A_t - P_t,$$

$$A_t = a + T_t + K_0, \quad t = 0, 1, 2.$$

Here K_0 is the permanent shock revealed at time 0 and T_t the temporary shock at time t. The expected value of A_t is a, which is large enough at $A_t > 0$ with certainty. K_0 and T_t are independent. The example also has a one-period interest rate of zero: $(1 + r) = R = 1$. From (2) and the definition of excess demand we have:

$$\begin{aligned} P_0 &= A_0 + I_0, \\ P_1 &= A_1 - I_0 + I_1, \qquad (16) \\ P_2 &= A_2 - I_1. \end{aligned}$$

These expressions are derived assuming that initial and final inventories are zero.

We solve this model recursively. At time 1 the information set is composed of K_0, T_1 and I_1. Then

$$E_1[P_2] = a + K_0 - I_1 = F_{1,2} \qquad (17)$$

and

$$P_1 = a + T_1 + K_0 - I_0 + I_1. \qquad (18)$$

Using the arbitrage relationship for positive inventories, (4) we find that positive I_1 implies that

$$I_1(1 + R) = (a + K_0)(1 - R) + (I_0 - T_1)R - c_1. \tag{19}$$

Thus I_1 will be positive or zero accordingly as the right-hand side of (19) is positive or nonpositive.

At time zero, information consists of T_0, K_0, and I_0. Then

$$I_1 = 1/2((I_0 - T_1) - c_1) \quad \text{if } I_1 > 0, \tag{20}$$

so that

$$I_1 > 0 \quad \text{iff } T_1 < I_0 - c_1. \tag{21}$$

These facts allow the deduction that $\partial I_0/\partial K_0 = 0$.

If $I_0 = 0$, $\partial I_0/\partial K_0 = 0$ immediately. If $I_0 > 0$, we have $P_0 R + c_0 = E_0[P_1]$. Let T_1^* be that which solves (19). Then $I_0 > 0$ implies

$$\begin{aligned}(a + K_0 + T_0 + I_0)R + c_0 \\ = \int_{T_1=-\infty}^{T_1=T_1^*} [(a + T_1 + K_0 - I_0) \\ + [(a + K_0)(1 - R) + (I_0 - T_1)R - c_1]/(1 + R)]f(T_1)dT_1 \\ + \int_{T_1=T_1^*}^{T_1=\infty} (a + T_1 + K_0 - I_0)f(T_1)dT_1.\end{aligned}$$

This equation determines I_0; no other endogenous variables appear. To see if I_0 depends on K_0, perform the comparative statics

$$\begin{aligned}RdK_0 = 0[(\partial T_1^*/\partial K_0)dK_0 + (\partial T_1^*/\partial I_1^*)dI_0] \\ + \int_{T_1=-\infty}^{T_1^*} [dK_0 - dI_0 + (1/(1 + R))((1 - R)dK_0 + dI_0)]f(T_1)dT_1 \\ + \int_{T_1=T_1^*}^{\infty} (dK_0 - dI_0)f(T_1)dT_1,\end{aligned}$$

so that $\partial I_0/\partial K_0 = 0$ if $R = 1$. That is, a permanent shock will have no impact on inventory holding in a zero interest rate economy. If the interest rate is positive, I_0 will depend on K_0, but the effect will be small. Now rewrite (18) in long form for $R = 0$:

$$P_1 = a + T_1 + K_0 - I_0 + \max[0, 1/2(I_0 - T_1 - c_1)]. \tag{22}$$

From (22) we can immediately distinguish the effects of a permanent from a temporary shock. First, in the event of a period 1 stockout, $\partial P_1/\partial K_0 = \partial P_1/\partial T_1$. But if I_1 is nonzero then K_0 has a larger impact on price at time 1 than does T_1. The intuition here is clear. Loosely speaking, K_0 is three times as big a shock as is T_1. Thus it will increase the value of the commodity more as long as the commodity is being stored. Similarly, (19) shows the effect of K_0 and T_1 on the probability of a stockout at time 1. The permanent shock K_0 adds as much to demand in period 1 as does the temporary shock T_1, but T_1 has a much larger coefficient on the RHS of (19). The permanent shock increases the probability of a stockout less, even though it shifts the present value of the commodity much more. As a result, temporary shocks rather than permanent ones create backwardation. Our formal theory presumed the existence of temporary shocks by assuming that A_t was not completely predictable.

The example shows the difference between permanent and temporary shocks to demand or supply of a storable commodity. A permanent shock raises the value of the commodity in all time periods, thus raising the commodity's price considerably but having little effect on backwardation. A temporary shock can have two different effects. If it does not cause a stockout, it is simply a smaller version of a permanent shock. It will then affect price only through its contribution to the present discounted value of the commodity. If a temporary shock causes a stockout, however, its effect on price now is as great as if it had shifted the demand curve equally in all future periods. Since price can reflect the transitory as well as the permanent value of the commodity, backwardations occur.

V. SOME PRELIMINARY EVIDENCE

The main empirical implication of this paper is given by equations (9), (14) and (15). Combined they imply that if the term structure of future prices exhibits backwardation, then prices are expected to rise by less than the carrying charges. Thus future markets backwardations predict capital losses to holding the commodity. According to the risk premium explanation, however, backwardations do not necessarily predict capital losses.

In this section we present very preliminary evidence supporting our main empirical proposition.

We gathered for four commodities (copper, pork bellies, frozen broilers and heating oil) spot and three months future prices. For each time period t we calculated whether there was a backwardation by calculating $B_t = P_t - F_{t,t+3}$. If $B_t > 0$, then there was a backwardation at time t. We also calculated the capital gains from holding the commodity for three periods: $G_t = P_{t+3} - P_t$. If $G_t < 0$, then an investor suffered a capital loss. Note that both B_t and G_t are defined as if carrying charges were zero.

Our evidence supporting the proposition that backwardations predict capital losses is presented in Tables I–IV. These tables present, for each commodity, a division of the sample into four categories. One category (presented in the northwest corner of each table) shows the number of observations which at time t did not show a backwardation (i.e., $P_t < F_{t,t+3}$) and on which there were capital gains ($P_{t+3} > P_t$) in the next three-month period. In the southeast corner of each diagram we present the number of observations for which there were both a backwardation and a capital loss. We observe that in all four commodities most of the observations appear at the diagonal cells. If at time t there was a backwardation, on average 85 percent of the observations showed a capital loss. In the same way, if at time t there was not a backwardation, more than 70 percent of the observations showed a capital gain.

Table I
Backwardation as a Prediction of Capital Gains and Losses
Copper*

	$P_{t+3} > P_t$	$P_{t+3} \leq P_t$		
$F_{t,t+3} > P_t$	48	26	74	Contangos
$F_{t,t+3} \leq P_t$	9	23	32	Backwardations
	57	49	106	
	Capital Gains	Capital Losses		

*Spot and futures prices on the New York CoMex, 1966–1983. These and all other market data from the Columbia University Center for the Study of Futures Markets Commodity database. Observations missing any of P_t, $F_{t,t+3}$, or P_{t+3} have been deleted. Observations are as of the last weekday of the month.

Table II
Backwardation as a Prediction of Capital Gains and Losses
Bellies*

	$P_{t+3} > P_t$	$P_{t+3} \leq P_t$		
$F_{t,t+3} > P_t$	33	14	47	Contangos
$F_{t,t+3} \leq P_t$	0	21	21	Backwardations
	33	35	68	
	Capital Gains	Capital Losses		

*Spot and futures prices for frozen pork bellies on the CME, 1966–1983.

Table III
Backwardation as a Prediction of Capital Gains and Losses
Broilers*

	$P_{t+3} > P_t$	$P_{t+3} \leq P_t$		
$F_{t,t+3} > P_t$	19	10	29	Contangos
$F_{t,t+3} \leq P_t$	8	26	34	Backwardations
	27	36	63	
	Capital Gains	Capital Losses		

*Spot and futures prices for ICED Broilers on the CBOT, 1974–1981.

Table IV
Backwardation as a Prediction of Capital Gains and Losses
Heating Oil*

	$P_{t+3} > P_t$	$P_{t+3} \leq P_t$		
$F_{t,t+3} > P_t$	23	2	25	Contangos
$F_{t,t+3} \leq P_t$	1	10	11	Backwardations
	24	12	36	
	Capital Gains	Capital Losses		

*Spot and futures prices on the New York Mercantile Exchange, 1978–1983.

This piece of evidence seems to suggest that (nominal) backwardations are good predictors of (nominal) capital losses, thus providing some support to the liquid stocks theory of backwardation.

VI. CONCLUSION

We have shown that backwardation is a normal feature of a risk-neutral economy. Storage has an asymmetric social constraint—negative storage is impossible. The dual to this constraint is that the storable commodity cannot be more valuable later than sooner, though it can be more valuable sooner than later even in equilibrium. Arbitrage in a risk-neutral world rules out contango (the reverse of backwardation), except for the costs-of-storage contango, but allows arbitrary backwardation.

A complete theory of equilibrium spot and futures prices clearly needs to treat risk aversion. We see the two explanations of backwardation, the one based on liquid stocks and the other on risk premia, as complementary. Our development of the liquid stock theory shows a crucial role for inventories and storage. Our conjecture is that a treatment of storage, stockouts and instantaneous supply in a risk-averse world would lead to a model in which the real risk premia in futures prices were endogenously time-varying. This paper has laid the groundwork for an empirical investigation of what fraction of normal backwardation is explained by risk premia, what by expected future spot prices. Our results imply that, under risk neutrality, spot prices should be unbiased predictors of future spot prices when there is no backwardation. When backwardation is present, spot prices should be upward biased. Thus a strong test between the two theories is available by splitting historical data into time periods with and without backwardation. Moreover, the preliminary empirical evidence presented here provides some support to the liquid stocks theory.

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