Duopoly Models with Consistent Conjectures

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The theory of oligopoly price is very sensitive to behavioral assumptions. Even given identical assumptions about costs and demand, different models can predict every price between marginal cost and monopoly. This paper selects a single oligopoly model, and thus predicts a single oligopoly price. The selection criterion is consistency of conjectures; each firm's conjectures about the way other firms react to it will be correct.

The two classical oligopoly theories, Bertrand and Cournot, make identical assumptions about costs and demand, but different assumptions about firm behavior. In Cournot equilibrium, each firm maximizes profit given the *quantity* of output other firms produce. In Bertrand equilibrium, each firm maximizes given the *prices* other firms charge. This difference in behavioral assumptions leads to a large divergence in predicted prices. Cournot predicts positive markups that decline as the number of firms increases, while Bertrand predicts marginal cost pricing even in duopoly. Clearly both models cannot be correct. Is their truth an empirical question, as recent work suggests?[1] This paper attempts to decide on theoretical grounds.

No attempt to decide among Bertrand, Cournot, and their more modern competitors can be based on mathematical correctness. Economic criteria must guide the decision. Oligopoly models are examples of what game theorists call Nash equilibrium. In them, every firm maximizes profits given the actions of all other firms. The mathematics does not care whether "actions" are defined to be prices (Bertrand), quantities (Cournot), or any other variables. Yet these distinctions are crucial to the economics of the situation. The notion of Nash equilibrium already entails one economic condition—individual rationality. This paper will determine the correct definition of actions by imposing a further economic condition—consistency of conjectures.[2]

The precise sense in which conjectures are to be consistent is this; the *conjectural variation* and the *reaction function* will be equated. The conjectural variation is the firm's conjecture about other firms' behavior. In Cournot, for example, each firm conjectures that all other firms' quantities are constant. The reaction function is the firm's actual behavior. It is the solution to the profit-maximizing problem, and tells what the firm will do as a function of all other firms' actions. Clearly, what the firm conjectures affects how it reacts. This paper will search for cases where conjectures and reactions are the same—where each firm's conjectures about other firms' reactions are perfectly correct, locally.[3]

Every notion of Nash equilibrium has the feature that, in equilibrium, each firm's beliefs about the *level* of all other firms' actions are confirmed. For example, in Cournot duopoly, each firm's equilibrium quantity is that one which induces the other firm to produce its equilibrium quantity. The firms are right in their beliefs, in Fellner's famous remark, but right for the wrong reason. That is, it is not actually true, as conjectured by the firm, that the other firm's quantity is a constant. The other firm's quantity depends nontrivially on ours—the reaction function does not have zero slope, although the conjecture does. This paper will find Nash equilibrium notions in which firms are right for

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[1]Two recent papers treating the oligopoly equilibrium notion as an empirical question are Elie Appelbaum and my own.

[2]A review of oligopoly models, of the idea that they can predict any prices, and of the notion of "conjecture" can be found in William Fellner or James Friedman.

[3]Similar notions of consistency have been taken up independently by other researchers. See John Laitner, David Ulph, and Morton Kamien and Nancy Schwartz.

the right reason. In equilibrium, they will be correct not only about the *levels* of one anothers' actions but also about the *functions* according to which they are reacting.[4] One might view this as a kind of rational expectations oligopoly theory. Consistency of conjectures makes the way firms react to one another endogenous by requiring that it be correct.

In the next section, a consistent conjectures equilibrium (*CCE*) is defined. A series of examples show what *CCE* price is under different assumptions about cost and demand. For example, with constant marginal costs the Bertrand conjectures (which imply marginal cost pricing) are consistent. A second section gives the central theorem of the paper: Under certain assumptions about cost and demand, the *CCE* exists and is unique. Uniqueness is a very important property—when the *CCE* is unique, consistency of conjectures solves the problem of too many oligopoly solution concepts by determining a single equilibrium price and quantity. Section III points out two ways in which increasing returns to scale could cause nonexistence of the *CCE*. Section IV investigates the role of the *CCE* when the conditions of the duopoly are set by firm's investments in capacity. It concludes that, with constant returns, capacity cannot serve as a barrier to entry. Defense of the *CCE* as a sensible way to solve the oligopoly problem is postponed until Section V.

I

This section establishes notation and defines a consistent conjectures equilibrium. In the classical case studied by Bertrand and Cournot (constant marginal cost) it is shown that the Cournot equilibrium is not a *CCE*. It is further shown that the Bertrand equilibrium is a *CCE* in this case. The section concludes by examining two examples in which the *CCE* lies between Bertrand and Cournot, depending on the cost and demand functions.

The quantity produced by firm i ($i=1,2$) is labelled q_i. The vector of both firms' outputs is called q:

$$q \equiv (q_1, q_2). \tag{1}$$

If the products are perfect substitutes, it will be convenient to define the industry quantity, Q:

$$Q \equiv q_1 + q_2. \tag{2}$$

Let us assume that the inverse demand functions are all defined. The notation for demand is

$$P_i = P_i(q_1, q_2). \tag{3}$$

When the products are perfect substitutes, (3) takes the form

$$P_i = P = P(Q). \tag{4}$$

The costs to firm i are given by the cost function $c_i(q_i)$.

Let us use the notational convention of Lester Telser and write all oligopoly solution concepts as if they had quantities as strategic variables. To do this, we adopt the idea of conjectural variation, r. Firm i acts as if it believes

$$\partial q_j / \partial q_i = r_{ij}(q_i) \text{ for } j=1,2; j \neq i. \tag{5}$$

Throughout, it shall be maintained the convention that firm j is the other firm when speaking about firm i. The observable implication of (5) is that firm i acts as if it believes that it faces a demand curve with slope

$$\frac{dP_i}{dq_i} = \frac{\partial P_i}{\partial q_i} + \frac{\partial P_i}{\partial q_j} r_{ij}(q_i). \tag{6}$$

Some examples may clear up the notion of conjectural variation. Let the two products be perfect substitutes. Then firm 1 acts as if it faces a demand curve with slope

$$dP/dq_1 = P'(Q)(1 + r_{12}(q_1)). \tag{7}$$

[4]The phrase "consistency of conjectures" or similar ones have sometimes been used to indicate correctness about the levels of strategic variables. This paper adapts the more common usage and calls correctness about levels "being in equilibrium."

Thus, if $r_{12}=0$, firm 1 is a Cournot player. If $r_{12}=-1$, firm 1 is a Bertrand player, since in this case total quantity, and therefore price, is conjectured to be a constant. If $r_{12}=1$, firm 1 acts like a colluder. In this case, the firm acts as if it can affect total output, but not its own market share. If $r_{21}=r_{12}$ in any of the above cases, then the reaction functions yield the named equilibrium concepts—Cournot, Bertrand, or Collusion. Of course, $r_{ij}(q_i)$ could be a more complicated function, and r_{12} need not equal r_{21}. What is important here is that assumptions about r are assumptions about the equilibrium concept.

We now examine the impact of the solution concept, that is, the r, on firm behavior. The profit function for firm i is

$$\Pi_i=P_i(q)q_i-c_i(q_i). \tag{8}$$

The corresponding first-order condition for a profit maximum is

(9)
$$0=q_i\left(\frac{\partial P_i(q_1,q_2)}{\partial q_i}+\frac{\partial P_i(q_1,q_2)}{\partial q_j}r_{ij}(q_i)\right)+P_i(q_1,q_2)-\frac{\partial c_i(q_i)}{\partial q_i},$$

which implicitly defines q_i in terms of the quantity produced by the other firm, q_j. The reaction function ρ is defined by

$$q_i=\rho_i(q_j) \text{ solves (9)}. \tag{10}$$

What function ρ is depends on r_{ij} as well as on the cost and demand functions. Note that in (9), the actual q_j enters even though firm i supposedly has conjectures about what q_j will be. These conjectures, therefore, are unlike those of a Stackelberg leader. In the present model, each firm does in fact react to the other, in a way that depends on conjectures. For any given conjectures, we could define an oligopoly equilibrium point, q^*, in the usual way:

$$\text{if } q_1^*=\rho_1(q_2^*) \text{ and } q_2^*=\rho_2(q_1^*), \tag{11}$$

then q^* is an equilibrium. I now extend this definition so that the conjectures are correct as well.

Definition: A consistent conjectures equilibrium is a pair of quantities q^*, and of conjectures $(r_{12}(q_1), r_{21}(q_2))$, such that

$$q_1^*=\rho_1(q_2^*),\ q_2^*=\rho_2(q_1^*), \tag{12}$$

and there is some $\varepsilon>0$, such that

$$r_{12}(q_1)=\frac{\partial\rho_2(q_1)}{\partial q_1} \quad \text{for all } q_1^*-\varepsilon<q_1<q_1^*+\varepsilon; \tag{13}$$

$$r_{21}(q_2)=\frac{\partial\rho_1(q_2)}{\partial q_2} \quad \text{for all } q_2^*-\varepsilon<q_2<q_2^*+\varepsilon. \tag{14}$$

The way to read (13) is: r_{12} is one's conjecture about two, ρ_2 two's actual behavior. In this definition (12) assures that both firms are correct about the *level* of one another's reaction functions. This is merely the usual Nash equilibrium condition. What (13) and (14) assure is that the firms are correct about the higher order derivatives as well. Perhaps the definition can be clarified by the familiar observation that the Cournot conjectures and reaction functions are not the same.

Example 1: Cournot equilibrium is not a *CCE* when marginal costs are constant, demand is linear, and products are perfect substitutes. Recall that the Cournot solution concept is defined by $r_{12}=r_{21}=0$. With constant marginal cost $\partial c/\partial q_i$, firm 1's profit is given by

$$\Pi_1=[P(Q)-\partial c/\partial q_i]q_1, \tag{15}$$

and the analog of (9) is

$$q_1P'(Q)+p(Q)-\partial c/\partial q_i=0. \tag{16}$$

An implicit differentiation gives the slope of 1's reaction function:

$$\partial\rho_1/\partial q_2=-P'(Q)/2P'(Q)=-1/2. \tag{17}$$

Exactly the same analysis could be carried out for firm 2, yielding

$$(18)\quad \partial\rho_2/\partial q_1 = -P'(Q)/2P'(Q) = -1/2$$

If the Cournot conjectures are to be consistent, (17) and (18) must be zero. Since they are equal to $-1/2$ everywhere, the Cournot conjectures are inconsistent, as is well known. Each firm assumes that the other firm's quantity is constant. Yet each firm, because of that assumption, has a reaction function which is not a constant. The firms optimal behavior differs from their assumption about one another's behavior.

The Cournot example makes clear that Cournot firms are not very sophisticated. I will not proceed by giving firms more sophisticated behavior: quite the reverse. We will look for those conjectures which are held by the firm to be certainly true, and which just happen to turn out to be correct. The spirit of this enterprise is therefore not one of giving firms discretion, but of removing their discretion by imposing correctness. Let us now search for the *CCE* under the same constant marginal cost assumption.

Example 2: Under the constant *mc* assumption, the Bertrand equilibrium is a *CCE* for any demand function $P(Q)$. Let the firms have identical linear conjectures with slope r. That is, $r_{12}=r_{21}=r$. Then the first-order condition for firm 1 is

$$(19)\quad (1+r)q_1P'(Q)+P(Q)-\partial c/\partial q_i=0.$$

and the slope of the reaction function is

$$(20)\quad \frac{\partial\rho_1}{\partial q_2} = -\frac{P'(Q)+(1+r)q_1P''(Q)}{(2+r)P'(Q)+(1+r)q_1P''(Q)}.$$

Note that if $r=-1$(Bertrand), then the reaction function has slope -1 as well. Since the situation is symmetric, firm 2's reaction function has the same slope. Therefore the Bertrand equilibrium is a *CCE* for this case.

The economic intuition of this example is straightforward. Recall that the Bertrand equilibrium has price equal to marginal cost. Suppose that one duopolist's behavior is: charge marginal cost and meet all demand. The only possible action for the other firm is marginal cost pricing. No action that attempts to reduce industry quantity can work, since the first firm will meet demand at marginal cost.

The conclusion that the Bertrand equilibrium is the *CCE* depends in a critical way on the cost and demand assumptions of the classical models. Let us now consider two examples in which those assumptions are relaxed. In the first of these, the constant marginal cost assumption is relaxed, and the marginal cost function is allowed to slope up. As the slope moves from horizontal to vertical, the *CCE* moves from Bertrand to Cournot.

Example 3: Let the total cost function be quadratic:

$$(21)\quad c(q_i)=c_0+c_1q_i+c_2(q_i)^2/2,$$

with c_0, c_1, c_2 all nonnegative. Further, let $P(Q)$ be linear with slope d. With a linear conjecture with slope r_{12}, firm 1's first-order condition is

$$(22)\quad P(Q)+d(1+r_{12})q_1 - (\partial c(q_1)/\partial q_1)=0.$$

The derivative of 1's reaction function is then

$$(23)\quad \partial\rho_1/\partial q_2 = -d/[(2+r_{12})d-c_2].$$

A straightforward calculation (shown in the Appendix) yields the slopes at the *CCE*. These equate the rs to the ρs.

$$(24)\quad r_{12}=r_{21} = -1+c_2[1-(1-4d/c_2)^{1/2}]/2d$$

As can be seen by inspection, the slopes of the consistent conjectures in this case lie between 0 (Cournot) and -1 (Bertrand). The *CCE* is determined by the ratio of the slopes of the marginal cost and demand functions. When that ratio is zero (constant marginal cost) the *CCE* is Bertrand equilibrium. As the ratio approaches infinity (vertical *mc*) the *CCE* approaches Cournot equilibrium. The latter polar case clarifies the intuition; if marginal cost curves are vertical, there are no sensible strategic variables except quantities.

In the next example, the *CCE* departs from Bertrand because the duopolists' products are not perfect substitutes. Again the intuition is clear. One would expect that the closer substitutes products are, the more competitive firm interactions are.

Example 4: Let the inverse demand function be linear and exhibit some product differentiation.

$$(25) \qquad \partial P_i/\partial q_k = d_{ik}; \quad i,k=1,2.$$

Marginal costs are constant at $\partial c/\partial q_i$. In this context, the Cournot conjectures continue to have slope zero. But the slope of the Bertrand conjectures is

$$(26) \qquad r_{ij} = -d_{jj}/d_{ji},$$

which is -1 only if the products are perfect substitutes. In the case of imperfect substitutes, therefore, the Bertrand equilibrium prices are above marginal cost. The *CCE* prices are yet higher. To see this, first calculate the first-order maximum condition for firm i:

$$(27) \quad 0 = P_i(q) - \partial c/\partial q_i + q_i(d_{ii} + d_{ij}r_{ij}).$$

The reaction function has slope:

$$(28) \quad \partial\rho_i/\partial q_j = -d_{ij}/(2d_{ii} + d_{ij}r_{ij}).$$

An easy calculation (see the Appendix) gives the *CCE* conjectures' slopes:

$$(29) \quad r_{ij} = \Big[-d_{ii}d_{jj} + \big[(d_{ii}d_{ij})(d_{ii}d_{jj} - d_{ij}d_{ji})\big]^{1/2}/d_{ij}d_{jj}\Big].$$

Note that the *CCE* conjectures do not involve square roots of negative numbers as long as both demand functions slope down in own price and the products are no more than perfect substitutes. Equation (29) has two interesting polar cases. First, if either d_{12} or d_{21} is zero, the consistent conjectures are those of Cournot. (And of Bertrand—without demand-side interaction, all oligopoly solution concepts are identical.) Second, if the determinant under the second radical is zero, that is, if the products are perfect substitutes, the *CCE* is Bertrand. In general, the *CCE* lies between Bertrand and Cournot, with the Jacobian determinant of the demand function determining exactly where.

II

This section investigates the (local) sense in which a general linear duopoly has a unique *CCE*. Uniqueness is important, because it shows that the *CCE* does determine a single duopoly equilibrium. Before stating the uniqueness theorem, let us look at one more example, which shows that the conjectures must be completely correct for the *CCE* to be unique.

Example 5: Suppose that firms have nonlinear conjectures ($r_{ij}(q_i)$ not a constant) but that we only require that their conjectures be linearly correct. This partial consistency imposes no restriction on the equilibrium prices and quantities. The example again uses the constant marginal cost case with perfect substitutes. Demand has slope d and cost c'. But now the conjectural variations are quadratic, with slopes:

$$(30) \qquad r_{ij}(q_i) = r_1 + r_2 \cdot (q_i - \theta),$$

where r_1, r_2, and θ are parameters. Then the slope of firm i's reaction function is

$$(31) \quad \partial\rho_i(q_j)/\partial q_j = -1/(2 + r_{ij}(q_i) + r_2 q_i).$$

The limited notion of consistency is this: at the equilibrium quantities, the slopes of the actual reaction functions are equal to the conjectured slopes. The second derivatives are left arbitrary.

With that notion, we can get limited consistency of any reaction slope between 0 and -1. First, set r_1 equal to the desired reaction slope. Set θ equal to the equilibrium quantity at that slope. That leaves this equation for the reaction slopes at those equilibrium quantities:

(32) $\partial \rho_i(\theta)/\partial q_j = -1/(2+r_1+r_2\theta)$

and then r_2 can be picked at will to make the reaction function have slope r_1.

The example clearly generalizes as far as desired. The firms could be incorrect about the forty-second derivative of the reaction function, rather than the second. Prices would still be arbitrary. The sense in which consistent conjectures are correct is therefore quite strong. The reaction functions are exactly the same functions as the conjectures, at least at the equilibrium point.

The assumptions needed for the uniqueness theorem should be familiar from remarks surrounding the examples:

ASSUMPTION 1: *The inverse demand functions are linear in the positive orthant and are truncated at the axes. In the interior of the positive orthant, the Jacobian of the demand system is negative semidefinite. It is negative definite unless the products are perfect substitutes.*

ASSUMPTION 2: *The total cost function is quadratic, positive-valued, increasing, and convex over positive quantities. That is, both total and marginal cost functions slope up.*

These assumptions combine the elements of departure from the classical case of examples 3 and 4.

ASSUMPTION 3: *Fixed costs are not so large as to swamp variable profits, nor are the firms' cost functions so different that one is dominated out of the market.* (This assumption will be made precise in the course of the proof. Without it, uniqueness would not be called into question, but existence would.)

THEOREM 1: *Under Assumptions* 1–3, *there is a CCE with linear conjectures. The CCE is unique in the class of polynomial conjectures.* (Proof is shown in the Appendix.)

Uniqueness in the class of polynomial conjectures is about as strong a result as can be gotten here. The reason for this is that conjectures about behavior far from the equilibrium point are irrelevant. If, say, doubling output were conjectured to cut profit by one-third instead of one-half, there would be no effect on behavior. Precisely the sense in which the *CCE* can be unique is local. The conjectures and the reaction function have the same Taylor series at the equilibrium point. Since the polynomials in the theorem are of arbitrarily large degree, this is what has been proven. For example, it has been shown that the equilibrium notion of Bertrand is the uniquely correct one in the classical case of constant marginal cost.

III

This section presents two examples of nonexistence of the *CCE* under increasing returns to scale. The first of these posits perfect substitutes, constant marginal costs, and nonzero fixed costs. The second example is constructed with a linear, downward-sloping marginal cost function.

When *mc* is constant and products are perfect substitutes, the *CCE* equilibrium concept will be Bertrand. Then equilibrium prices will be equal to *mc*—an unfortunate outcome for firms with fixed costs. Since revenue just covers variable cost, profits will be negative. Exit should follow. This *CCE*, which was an equilibrium given only local information, is not an equilibrium when the total conditions are taken into account. Thus, in this case, there will be no *CCE*.

Example 6: The cost function is the quadratic one of example 3, above. But here $c_0=0$, $c_1>0$, $c_2<0$, so that *mc* slopes down. $P(Q)$ is linear with slope d. The inverse demand function cuts the *mc* curve from above in the positive orthant. I begin by looking for a *CCE* with linear conjectures. The derivatives of the reaction functions are exactly as in example 3, above:

(33) $\partial \rho_1/\partial q_2 = -d/(d(2+r_{12})-c_2)$;

(34) $\partial \rho_2/\partial q_1 = -d/(d(2+r_{21})-c_2)$.

Imposing the conditions of the *CCE* and substituting (33) into (34) yields

$$(35)\quad r_{12}\left[2-\frac{1}{2+r_{12}-c_2/d}-\frac{c_2}{d}\right]=-1.$$

The quadratic equation implied has determinant

$$(36)\quad (2-c_2/d)^2-4<0,\ \text{if}\ c_2<0.$$

Thus no linear *CCE* exists for $c_2<0$.[5] Since Lemma 2 (see the Appendix) shows that linear duopolies like this one have linear *CCE*'s if they have any polynomial ones, no polynomial *CCE* exists for this technology.

It is easy to understand the nature of this nonexistence proof, and of the comparable result when the technology has fixed costs and constant marginal costs. In these instances, global increasing returns reign. Any duopoly equilibrium is dominated in cost terms by some single firm outcome. Hence it is the theory of entry, not of duopoly, which determines price. Monopoly equilibria do exist. Thus only equilibria with the cost minimizing number of firms in operation exist.

IV

This section presents an example of capacity investment oligopoly. In it, firms invest in fixed capital in advance of the market period. Then it is the *SR* cost function which is relevant to the market equilibrium. Since the market equilibrium is a *CCE*, there is no intertemporal inconsistency in firms' beliefs about one anothers' behavior. The example explores the nature of capacity as a barrier to entry (see Michael Spence) under the assumption that the post-entry duopoly solution concept is a *CCE*. It clearly shows that capacity alone is not a sufficient barrier with a constant returns Leontief technology.

Example 7: Producers must invest in fixed capital in advance of the market period, and cannot alter their capacity during the market period. The overall production technology is Leontief:

$$(37)\quad q_i=min[L_i/\lambda, K_i/\kappa],$$

where L_i is labor, K_i capital in place, and λ and κ are constants. Label firm i's capacity by $k_i=K_i/\kappa$ Then the short-run (fixed K_i) marginal cost is

$$(38)\quad srmc_i=\begin{cases} c & 0\leqslant q_i<k_i \\ \infty & q_i\geqslant k_i \end{cases}$$

where c is λ times the wage rate. Long-run total costs include a charge for capital, of course. Let f be κ times the interest rate. Then,

$$(39)\quad lrmc_i=c+f.$$

As before, demand is linear: $P=A+d(q_1+q_2)$.

The equilibrium concept has two stages, capacity and market. By imposing *CCE* on the market equilibrium, we insure that perfect foresight at the capacity investment stage involves no inconsistency. As a first step, let us calculate the market *CCE* as a function of capacities. The capacity constraint changes the firms' maximization problems, since profit maxima can come either at an interior solution or at the capacity constraint. The Kuhn-Tucker conditions for a profit maximum are

$$(40)\quad q_i d(1+r_{ij})+P(Q)-c\geqslant 0,$$
$$[q_i d(1+r_{ij})+P(Q)-c][k_i-q_i]=0.$$

In working toward calculation of the *CCE*, it will be useful to have a few points on the mapping from conjectures to reaction slopes on hand, as in Table 1. The table shows what kind of *CCE*s are possible. Both firms could be unconstrained; in this case they will be Bertrand players, as example (2) showed. This occurs only if industry capacity is sufficient to set demand price to c. Alternatively, both firms could be constrained; this occurs when their capacities are both so small that the constraint binds with the (correct) con-

[5]Note that the determinant is positive if $2d<c_2<0$. But then the inverse demand function cuts mc from below and the second-order conditions cannot hold.

TABLE 1

Constrained?	Conjecture	Reaction
yes	any	0
no	0	$-\frac{1}{2}$
no	-1	-1

jecture of zero. Lastly, one firm could be constrained (with conjecture $-1/2$) and the other unconstrained (with conjecture 0).

Manipulation of the first-order conditions reveals the regions in capacity space in which each of these equilibria can obtain. In Figure 1, region I has both firms constrained, region II, neither, while in regions III and IV firms one and two only are constrained, respectively. In region V there are two equilibria. Either firm 1 or firm 2 can be constrained but not both.

The nonuniqueness of equilibrium in region V is not disturbing in the context of the two-stage game. Any notion of individual rationality suggests that no firm will invest in capacity and then become unconstrained. Therefore the interior of regions III, IV, and V can be ruled out. On the border, of course, the profits are the same whether the firm is viewed as just constrained or just not. A problem might arise if each firm took the (optimistic) view that it was going to be the constrained firm, even if it had the larger capacity. These mutually inconsistent expectations could lead to inconsistent two-stage equilibria.

More interesting is the light in which this example places the use of capacity as a barrier to entry. How does an incumbent firm's excess capacity serve as a signal of willingness to compete *ex post* entry? If the post-entry solution concept is the *CCE*, capacity does not have the desired effect. With constant returns to scale, the incumbent cannot find a capacity which induces the potential entrant to stay out. This is true even if we assume that the incumbent is a Stackelberg leader in the capacity game. The key to the proof is the nature of the *CCE* when either firm's capacity is small.

To see that capacity cannot be a barrier, assume that firm 1 is the incumbent. Divide the problem of the entrant, firm 2, into two cases depending on firm 1's capacity.

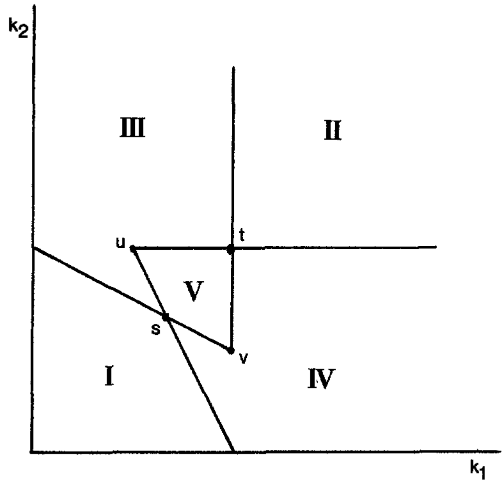

FIGURE 1. $s=\left[\frac{A-c}{-3d},\frac{A-c}{-3d}\right]$; $t=\left[\frac{A-c}{-2d},\frac{A-c}{-2d}\right]$; $u=\left[\frac{A-c}{-4d},\frac{A-c}{-2d}\right]$; $v=\left[\frac{A-c}{-2d},\frac{A-c}{-4d}\right]$.

Case I: $k_1<-(A-c)/2d$. In this case, if firm 2 enters at small enough capacity, the *CCE* will be in region I, both firms constrained. Therefore, if firm 1 was making positive profits, firm 2 can find a small enough capacity to make positive profits as well.

Case II: $k_1 \geqslant -(A-c)/2d$. In this case, firm 1 will have excess capacity even after entry at small k_2. In region IV, where firm 2 is capacity constrained, firm 1 is not. In that region, firm 1 (correctly) conjectures that firm 2's output is constant, while firm 2 knows r_{21} to be $-1/2$. Then the equilibrium market values will be

$$(41)\qquad q_2=k_2, q_1=-(A-c)/2d-k_2/2;$$

$$(42)\qquad P=A-(A-c)/2+dk_2/2;$$

$$(43)\qquad \Pi_2=\left[(A-c)/2+dk_2/2-f\right]k_2.$$

Equation (43) says that the condition for firm 2 to be unable to find a capacity investment which yields positive profit is

$$f>(A-c)/2. \tag{44}$$

But this is exactly the condition that firm 1's monopoly price not cover capital charges should entry be deterred. Thus deterring entry cannot be profitable.

This example shows that excess capacity alone is not a barrier to entry when all players correctly anticipate the post entry equilibrium. This is true even though the incumbent firm can alter the "rules of the game" as well as its "initial conditions" in Avinash Dixit's useful distinction. It is clear from the proof that this result leans heavily on the assumption of constant returns down to arbitrarily small scale.

V

This section defends consistency of conjectures as a reasonable restriction on the oligopoly solution concept. The defense, suprisingly enough, has nothing to do with dynamics. Indeed, the problem of constructing an informationally consistent oligopoly is a formidable one, as yet unsolved. The first half of this section treats dynamic arguments; the results are quite negative. The second half treats the comparative statics of equilibrium; they lead naturally to the *CCE*.

One argument against some oligopoly solution concepts is that they have "inconsistent dynamics." Although I will not use the dynamic inconsistency argument, it is instructive to step through it for Cournot equilibrium. Begin with an arbitrary quantity for firm one, say q_1. Let firm two take q_1 as given and move to its profit-maximizing output, $q_2=\rho_2(q_1)$. Now firm one will move to $q_1=\rho_1(q_2)\ldots$. Continue until convergence at the equilibrium quantities. At every step, each firm has maximized profit, taking the quantity of the other firm as given. Yet the other firm's quantity has in fact changed; the firm should notice this and take advantage of the information. A possible argument for the *CCE* arises from this dynamic inconsistency. If firms' conjectures are everywhere correct, they will be confirmed by these dynamics.

The inconsistent dynamics argument is unsatisfactory because it confuses statics and dynamics. Is it an argument against Cournot equilibrium or against the particular dynamic used to reach it? Economists routinely object to similar cobweb dynamics for perfect competition on rational expectations grounds. Perhaps here, as there, the fault could lie in the dynamic rather than in the equilibrium notion. It would be desirable[6] to construct a better argument, a rational expectations oligopoly, in which dynamic considerations affect the static equilibrium concept. But it is not necessary.

Fortunately, it is not necessary to work out the disequilibrium dynamics of oligopoly to defend the *CCE*. The comparative statics of equilibrium give firms enough information to recover one anothers' behavior. Suppose that some variable exogenous to the oligopoly (say, costs, the location of the demand curve) is changed. Equilibrium prices and quantities will change whatever the nature of the equilibrium concept. Suppose that firms learn nothing about one anothers' behavior from the dynamic process by which the new equilibrium is obtained. They can still learn one anothers' reactions from the location of the new equilibrium. A Cournot firm, knowing the costs, the demands, and that it has reaction slope $-1/2$ can easily learn from the movement of market price the one thing it does not know—the slope of the other firm's reaction. The natural experiment, the

[6]It is quite difficult. The discussions of oligopoly by Takashi Negishi and by D. W. Bushaw and Robert W. Clower (ch. 7) as well as the "conjectural equilibria" of Frank Hahn and Jose Trujillo will be useful in attacking the out-of-equilibrium dynamics. The theory will have to treat several thorny issues. One is a kind of higher-order inconsistency. Suppose constant marginal cost duopolists begin with Cournot ($r=0$) conjectures. Each then observes that the other has a reaction with slope $-1/2$, and updates its conjecture to have this slope. This gives them (optimally: see (23), above) reactions with slope $-2/3$. And this process will continue until they converge to the consistent Bertrand conjectures. Note that the same dynamic inconsistency argument raised against Cournot dynamics can be raised here. The firms are acting as if one anothers' *reactions* are fixed. Why do they not notice that they are changing? In light of this line of argument, I am not sanguine about the possibilities of a fully consistent oligopoly dynamic.

movement of exogenous variables, can reveal to firms that their conjectures are inconsistent. This argument does not depend in any critical way on the belief that the equilibrium comparative statics "actually happen." Consider a two-stage oligopoly like the one in the last section. Firms must first pick capacity, and then, with capacity fixed, price or output. At the first stage the only reasonable expectations to give firms are the correct ones. They should be able to calculate their eventual profits as a function of their capacities. Performing that calculation requires knowledge of the second-stage equilibrium concept. Are we to assume, then, that at the second stage they forget their knowledge of one anothers' behavior? There will be no intertemporal inconsistency in their knowledge if the market equilibrium notion is the *CCE*.

The sense of the *CCE* is then this. If firms have inconsistent conjectures and it is possible for them to learn how their industry reacts to exogenous shocks, they will learn that their conjectures are wrong. If they have consistent conjectures, nothing in the comparative statics of equilibrium will reveal those conjectures to be wrong. By what dynamic process the conjectures will come to be consistent is an unsolved problem, as is the possibility of an informationally consistent, stable dynamic for oligopoly prices and quantities.

VI. Conclusion

This paper has solved, in one sense, "the oligopoly problem." The indeterminacy of duopoly price due to a multiplicity of solution concepts has been removed. The cost of a determinate price is the denial of all discretion in firm behavior.[7] Whether this cost is justified depends on the reasonableness of the conditions imposed.

The reasonableness of the *CCE* as such a condition was discussed in the last section. A second defense can be constructed from the solution yielded by the *CCE*. When products are perfect substitutes and marginal costs are constant, pricing will be competitive. When the marginal cost function slopes up or when products are less perfect substitutes, pricing becomes less competitive. This, I think, is a quite intuitive theory of competition. Competition comes about when increases in quantity have no adverse affects on costs and when products are close substitutes.

Related results hint at but do not resolve a connection between increasing returns to scale and entry. First, in two cases of increasing returns, there is no duopoly *CCE*. Second, capacity cannot serve as a barrier to entry when returns are constant down to arbitrarily small scale.[8] These results seem to point at a theory in which entry is determined by cost-minimizing market structure. More work on this appears warranted.

Appendix: Proof of Theorem 1

Theorem 1 can be proved in two lemmas. The first shows that there is exactly one *CCE* under the assumption that conjectures are linear. The second shows that, when the cost and demand systems are linear, any polynomial conjecture must be linear to be correct.

Recall some notation:

(A1) $$d_{ij} \equiv \partial P_i / \partial q_j,$$

$$c_i(q_i) = c_{i0} + c_{i1} q_i + c_{i2}(q_i)^2/2.$$

(A2) $\rho_i(q_j)$ is firm i reacting to j.

(A3) $r_{ij}(q_i)$ is i's conjecture about j.

LEMMA 1: *Under the assumptions of Theorem 1, there is exactly one CCE with linear conjectures.*

[7] A discussion of the role of discretion in the theory of the firm in general and in industrial organization in particular can be found in Donald Hay and Derek Morris.

[8] Some earlier readers of this paper feel that this suggests that the *CCE* is a "perfectness" notion, alluding to the work of Thomas Marschak and R. Selten. I can see no more than analogy in this, although the suggestion is intriguing.

PROOF:

The first-order conditions are

$$(A4)\quad 0=P_i-\partial c_i(q_i)/\partial q_i+(d_{ii}+d_{ij}r_{ij})q_i; \qquad i=1,2.$$

The slopes of the reaction functions are

$$(A5)\quad \frac{\partial \rho_i(q_j)}{\partial q_j}=\frac{-d_{ij}}{2d_{ii}-c_{i2}+d_{ij}r_{ij}}; \qquad i=1,2\ j\neq i.$$

Note that the linear economic structure implies that the ρ_{ij} are constant if the r_{ji} are. The *CCE* is defined by setting $\rho_{21}=r_{12}$, $\rho_{12}=r_{21}$. This leaves a quadratic equation in r_{12} and r_{21}. The roots are

$$(A6)\quad r_{ij}=-\alpha\pm\left[(\alpha)(\alpha-4d_{12}d_{21})\right]^{1/2} \Big/ \left[2d_{ij}(2d_{jj}-c_{j2})\right];$$

$$(A7)\quad \text{where } \alpha\equiv(2d_{11}-c_{12})(2d_{22}-c_{22}).$$

The numbers under the radicals are always nonnegative. $\alpha>0$ since $d_{ii}<0$, $c_{i2}\geqslant 0$. $\alpha>4d_{12}d_{21}$ since the products are assumed to be no more than perfect substitutes.

Equation (A6) seems to imply that there are two *CCE*s. (Not four, since taking the positive radical in r_{12} implies taking it in r_{21}, as well.) One of these is economically meaningless, since it occurs at negative quantities. The true *CCE* is found by taking the positive radical. To see this, substitute (A6) back into the first-order condition (A4):

$$(A8)\quad 0=P_i-\partial c_i(q_i)/\partial q_i + \left[d_{ii}+\left\{-\alpha\pm\left[(\alpha)(\alpha-4d_{12}d_{21})\right]^{1/2}\right\} \Big/ \left[2(2d_{jj}-c_{j2})\right]\right]q_i.$$

Since the markup $P_i-\partial c_i(q_i)/\partial q_i$ is always positive, the sign of q_i depends only on the bracketed term. Expanding the bracket yields

$$(A9)\quad d_{ii}-\frac{2d_{ii}-c_{i2}}{2} \pm\frac{2d_{ii}-c_{i2}}{2}\left(\frac{\alpha-4d_{12}d_{21}}{\alpha}\right)^{1/2}.$$

Since $d_{ii}<0$, $c_{i2}\geqslant 0$, (A6) is negative if and only if the positive radical is taken. Hence $q_i>0$ only if the positive radical is taken.

It should be mentioned at this point that, in the definition of the *CCE*, we have not allowed firms to shut down. Using only local information, we have no way of telling whether price is over average cost. There are two ways for the total condition to fail. One is if the fixed costs are so high that they overwhelm operating profits. This can clearly be dealt with only by calculating operating profits and assuming fixed costs smaller. The other difficulty arises when the firms are close substitutes and one of them has a cost advantage. For example, when *mc* is constant, and the products perfect substitutes, both firms must have the same *mc* or there is no equilibrium. This, too, is dealt with by direct ruling out under Assumption 3.

The exact assumptions on technology and demand made in Assumption 3 can now be spelled out explicitly. First, let the reaction slopes take on their *CCE* values:

$$(A10)\quad r_{ij}=-\alpha+\left[(\alpha)(\alpha-4d_{12}d_{21})\right]^{1/2} \Big/ \left[2d_{ij}(2d_{jj}-c_{j2})\right].$$

Then write out the equations defining the equilibrium quantities:

(A11)

$$(2d_{11}+d_{12}r_{12}-c_{12})q_1+d_{12}q_2=c_{11}-d_{10}$$

$$d_{21}q_1+(2d_{22}+d_{21}r_{12}-c_{22})q_2=c_{21}-d_{20};$$

the condition for q_i to be positive is then

$$(A12)\quad \left[2d_{jj}+d_{ji}r_{ji}-c_{j2}\right](c_{i1}-d_{i0}) -d_{ij}(c_{j1}-d_{j0})<0.$$

This condition will be satisfied whenever i's zero-quantity markup is not too much less than j's, or when i's demand is suitably independent of j's (in the sense that d_{ij} is small). The other half of Assumption 3 is that no shutdown occur. Using (A4), the condition that firm i's price exceed average cost is

$$\text{(A13)} \qquad mc - ac > q_i[d_{ii} + d_{ij}r_{ij}].$$

Since the bracketed term is negative, the right-hand side is as well. Thus price can fall below average cost only when average cost exceeds marginal in equilibrium. Since average cost is not falling under Assumption 2, this is only the assumption that the fixed costs are not too large.

LEMMA 2: *In the linear situation of theorem* 1, *any CCE with polynomial conjectures has linear conjectures. This is a straightforward proof by contradiction. Assume that* $r_{12}(q_1)$ *and* $r_{21}(q_2)$ *are polynomials of arbitrarily large order, say of order* k. *I reproduce the first-order condition*:

$$\text{(A14)} \quad 0 = P_i - c'_i(q_i) + q_i(d_{ii} + d_{ij}r_{ij}(q_i)) \qquad i = 1,2 \quad j \neq i.$$

The reaction function is defined by solving (A9) for q_i in terms of q_j. The slope of the reaction function is

$$\text{(A15)} \quad \partial \rho_i(q_j)/\partial q_j = -d_{ij}/(2d_{ii} - c_{i2} + d_{ij}r_{ij}(q_i) + d_{ij}q_i \partial r_{ij}(q_i)/\partial q_i).$$

The denominator of the right-hand side (A14) is a kth order polynomial in q_i. Firm i's reaction function has been assumed to be a polynomial in q_j. Therefore the left-hand side of (A14) is a $k-1$th order polynomial in q_j. Clearly there is no ε neighborhood on which these two assumptions are consistent unless $k=0$, the conjectures are linear.

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